Section 2.1 (Complex Numbers)

In this section we travel to an imaginary place called the ***complex number system***

- The imaginary unit i is used to denote the square root of -1 ($i^2 = -1$ and $\sqrt{-1} = i$)

Using i, we can express the square root of any number ($\sqrt{-25} = \sqrt{-1(25)} = \left(\sqrt{-1}\right)\sqrt{25} = 5i$)

The complex number system contains all real numbers and can be written as **a + bi**

Examples: The following are examples of complex numbers

– 4 + 6i

a, the real part is -4

b, the imaginary part is 6

2i = 0 + 2i

a, the real part is 0

b, the imaginary part is 2

3 = 3 + 0i

a, the real part is 3

b, the imaginary part is 0

Complex numbers can be manipulated as follows (given a + bi and c + di as complex numbers)

- **a + bi = c + di** if and only if the real parts are equal and the imaginary parts are equal (a=b & c=d)
- sum => add the real parts and then add the imaginary parts => **(a+bi) + (c+di) = (a+c) + (b+d)i**
- difference => same as sum but with subtraction => **(a+bi) – (c+di) = (a-c) + (b-d)i**

Examples: Add / Subtract

(5 + 2i) + (4 – 3i) =

6i – (2 – i) =

(–2 – 4i) + (–3) =

(3 + 4i) – (2 + 3i) =

Multiplying complex numbers is similar to polynomials, replacing occurrences of $\mathbf{i^2}$ with **-1**

Examples: Multiply (remember the FOIL method?)

– 5i * 3i =

– 2i (6 – 2i) =

(3 – 4i)(6 + i) =

$\mathbf{(1 - 2i)^2 =}$

(6 + 5i)(6 – 5i) =

Complex numbers **(a + bi)** and **(a – bi)** are called ***complex conjugates*** of each other and

$\mathbf{(a + bi)(a - bi) = a^2 + b^2}$ (real)

$\mathbf{(a - bi)(a + bi) = a^2 + b^2}$ (real)

We use complex conjugates to divide a complex number (multiply top and bottom by the conjugate of the denominator to eliminate imaginary number in the denominator)

Book problems: 1,3,11,15,19,25,27,25,27,43,45,49,53

Examples: Divide (write answer in the standard form a + bi)

$$\frac{3+i}{2-3i} \qquad\qquad \frac{7+4i}{2-5i}$$

Keep in mind that the product rule for radicals doesn't necessarily hold true for ***imaginary numbers*** (when multiplying, always write numbers in terms of i first)

~~$\sqrt{-4}\sqrt{-9} = \sqrt{(-4)(-9)} = \sqrt{36} = 6$ NO~~

$\sqrt{-4}\sqrt{-9} = 2i(3i) = 6i^2 = 6(-1) = -6$ YES

When performing operations with square roots of negative numbers, begin by expressing all square roots in terms of i

Examples: Perform the indicated operations and write the result in standard form

$$\left(-2+\sqrt{-3}\right)^2 \qquad\qquad \frac{-14+\sqrt{-12}}{2}$$

Recall that a quadratic equation can be expressed in the general form as $ax^2 + bx + c = 0$, $a \neq 0$. Also, all quadratic equations can be solved by the ***quadratic formula***

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This sometimes results in imaginary numbers that are complex conjugates.

Example: Solve using the quadratic formula

$$x^2 - 2x + 2 = 0 \qquad\qquad 7x^2 = 2x - 1$$

Book problems: 1,3,11,15,19,25,27,25,27,43,45,49,53