

Section 2.1 (Complex Numbers)

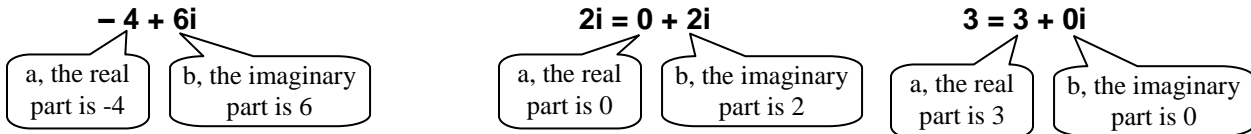
In this section we travel to an imaginary place called the **complex number system**

- The imaginary unit i is used to denote the square root of -1 ($i^2 = -1$ and $\sqrt{-1} = i$)

Using i , we can express the square root of any number ($\sqrt{-25} = \sqrt{-1(25)} = (\sqrt{-1})\sqrt{25} = 5i$)

The complex number system contains all real numbers and can be written as $a + bi$

Examples: The following are examples of complex numbers



Complex numbers can be manipulated as follows (given $a + bi$ and $c + di$ as complex numbers)

- $a + bi = c + di$ if and only if the real parts are equal and the imaginary parts are equal ($a=b$ & $c=d$)
- sum \Rightarrow add the real parts and then add the imaginary parts $\Rightarrow (a+bi) + (c+di) = (a+c) + (b+d)i$
- difference \Rightarrow same as sum but with subtraction $\Rightarrow (a+bi) - (c+di) = (a-c) + (b-d)i$

Examples: Add / Subtract

$$(5 + 2i) + (4 - 3i) =$$

$$6i - (2 - i) =$$

$$(-2 - 4i) + (-3) =$$

$$(3 + 4i) - (2 + 3i) =$$

Multiplying complex numbers is similar to polynomials, replacing occurrences of i^2 with -1

Examples: Multiply (remember the FOIL method?)

$$-5i * 3i =$$

$$-2i(6 - 2i) =$$

$$(3 - 4i)(6 + i) =$$

$$(1 - 2i)^2 =$$

$$(6 + 5i)(6 - 5i) =$$

Complex numbers $(a + bi)$ and $(a - bi)$ are called **complex conjugates** of each other and

$$(a + bi)(a - bi) = a^2 + b^2 \text{ (real)}$$

$$(a - bi)(a + bi) = a^2 + b^2 \text{ (real)}$$

We use complex conjugates to divide a complex number (multiply top and bottom by the conjugate of the denominator to eliminate imaginary number in the denominator)

Examples: Divide (write answer in the standard form $a + bi$)

$$\frac{3+i}{2-3i}$$

$$\frac{7+4i}{2-5i}$$

Keep in mind that the product rule for radicals doesn't necessarily hold true for **imaginary numbers** (when multiplying, always write numbers in terms of i first)

~~$$\sqrt{-4}\sqrt{-9} = \sqrt{(-4)(-9)} = \sqrt{36} = 6 \text{ NO}$$~~

$$\sqrt{-4}\sqrt{-9} = 2i(3i) = 6i^2 = 6(-1) = -6 \text{ YES}$$

When performing operations with square roots of negative numbers, begin by expressing all square roots in terms of i

Examples: Perform the indicated operations and write the result in standard form

$$(-2 + \sqrt{-3})^2$$

$$\frac{-14 + \sqrt{-12}}{2}$$

Recall that a quadratic equation can be expressed in the general form as $ax^2 + bx + c = 0$, $a \neq 0$. Also, all quadratic equations can be solved by the **quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This sometimes results in imaginary numbers that are complex conjugates.

Example: Solve using the quadratic formula

$$x^2 - 2x + 2 = 0$$

$$7x^2 = 2x - 1$$