<u>**Section 2.3**</u> <u>(Polynomial Functions and Their Graphs)</u>

(Remember that you can always pick a point for x and solve to help graph)

The function defined by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

is a ***polynomial*** of ***degree n*** (the largest of the exponents) if n (the ***exponents***) are ***nonnegative integers*** and a_n , a_{n-1} , … a_2 , a_1 , a_0 (the coefficients) are real numbers.

The number a_n , the coefficient of the variable to the highest power, is called the ***leading coefficient***

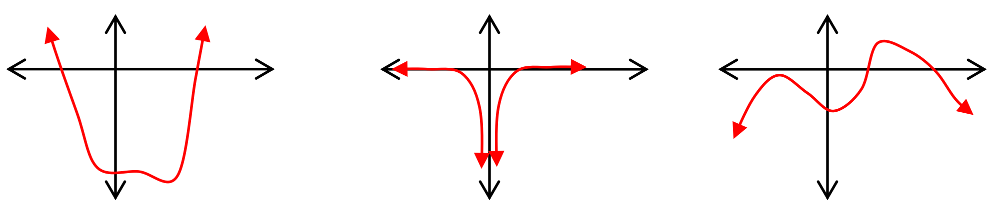
<u>Examples</u>: Determine if the following are polynomials. If so, state the degree. If not, tell why.

$f(x) = -6x + x^9$ $\qquad$ $g(x) = 6x - \frac{1}{x}$ $\qquad$ $h(x) = x^2 + 2x + x^{1/2} - 4$

$f(x) = 3x + 4$ $\qquad$ $g(x) = 7$ $\qquad$ $h(x) = \sqrt{3}x + \sqrt{x}$

Polynomial functions of degree 2 or higher have graphs that are smooth (rounded curves – no sharp corners) and continuous (no breaks and can be drawn without lifting pencil / pen) – note: what about degree 1 and degree 0?

<u>Examples</u>: Are the following graphs of polynomial functions?



Consider the trend of $f(x) = 2x^4 - 80x$

Trend	x	$f(x) = 2x^4 – 80x$	Result
x is small (all terms have an impact)	0	f(x) = 0 – 0 = 0	Graph falls from 0 to negative
	2	2(16) – 80(2) = -128	
x increases slightly (all terms still have some impact)	5	2(625) – 80(5) = 625	Graph rises right
	6	2(1296) – 80(6) = 2112	
x is large (the leading term dominates like the Hokies in football)	100	2(1000000000) – 80(100) = 199992000	Graph rises steeply
	…	Really big number	

As x increases / decreases without bound, the graph of the polynomial function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

eventually rises or falls depending on the leading term (term with the largest degree – the coefficient and the variable part).

The key is to look at the degree of the variable and the sign of the coefficient in the leading term

- Degree n is odd
  - Leading coefficient is positive => Graph falls to the left and rises to the right
  - Leading coefficient is negative => Graph rises to the left and falls to the right
- Degree n is even
  - Leading coefficient is positive => Graph rises on both sides
  - Leading coefficient is negative => Graph falls on both sides

Book problems: 1,5,7,13,14,19,21,23,27,29,31,41,43,55

Examples: Look at the chart on page 304 and use the Leading Coefficient Test to determine the end behavior of the graph of the following

$f(x) = 3x^7 – 2x^5 – 4x^2 + 6$ $g(x) = 8x^6 + 4x^5 + 8$ $h(x) = -4x^4 + 3$

The values of x for which the polynomial function f(x) is equal to 0 are called the ***zeros*** of f (also called the ***roots*** or ***solutions*** of f). Each real root is an x-intercept of the graph of the polynomial function.

Example: Find all zeros of…

$f(x) = x^3 + 2x^2 – 4x – 8$ $g(x) = x^4 – 4x^2$

In the 2nd example just shown, the factorization was $g(x) = x^2(x + 2)(x – 2)$. The factor x occurs twice and the factors (x + 2) and (x – 2) occur once. The number of times a factor occurs gives the ***multiplicity*** of that zero. So our solution 0 has multiplicity 2 and our solutions 2 and -2 have multiplicity 1.

If a zero (solution) has ***even multiplicity***, the graph touches the x-axis at this point and turns around. If a zero (solution) has ***odd multiplicity***, the graph crosses the x-axis at this point.

Example: Find the zeros / the multiplicity of each zero, and state the behavior of the graph at each zero

$g(x) = x^4 – 4x^2$ $f(x) = -4(x + ½)^2(x – 5)^3$

You can use the facts learned in this section and other steps mentioned (pg. 310) to sketch graphs of polynomial functions (note additional steps of finding y-intercept by finding f(0) and determining possible symmetry by looking at f(-x) = f(x) : y-axis symmetry, f(-x) = -f(x) : origin symmetry)

Examples: After reviewing the chart and steps on pg. 310, use those steps to graph $x^3 – 3x^2$ on the board

Book problems: 1,5,7,13,14,19,21,23,27,29,31,41,43,55