Section 2.5 (Zeros of Polynomial Functions)

The Rational Zero Theorem (pg. 327) allows us to make a list of all possible zeros (solutions) of a polynomial function with integer coefficients

Possible rational zeros = $\dfrac{\text{Factors of the constant term (on the end)}}{\text{Factors of the leading coefficient (the beginning)}}$

Example: List all possible rational zeros for $f(x) = x^3 + 2x^2 - 5x - 6$

Constant term = Factors =

Leading coefficient = Factors =

Possible zeros = { }

Given those rational zeros, we can then use synthetic division to determine a legitimate zero for the function.

Example: Test the above possible zeros using synthetic division to find an actual zero

1	1	2	-5	-6

Since _____ is a zero / solution (because of the remainder of 0), then x - _____ = **x + 1** is a factor or

$$f(x) = x^3 + 2x^2 - 5x - 6 = (x + 1)(\mathbf{x^2 + x - 6}) = (x + 1)(x + 3)(x - 2)$$

--- using results from synthetic division ---

Therefore the solution set of $f(x) = x^3 + 2x^2 - 5x - 6$ or the zeros of the given f(x) are { }

Example: Solve…

$x^3 + 8x^2 + 11x - 20 = 0$

$x^3 + x^2 - 5x - 2 = 0$

Example: Find the zeros of $f(x) = x^4 - 6x^3 + 22x^2 - 30x + 13$

Book problems: 1,5,6,9,11,17,23,25,27,33,35,39

Properties of roots of polynomial equations:

1. A polynomial of degree n has n roots (counting multiple roots separately)
2. If a + bi is a root to a polynomial equation ($b \neq 0$), then the conjugate a – bi is also a root

Notice in the above example, we had solutions of $\{1, 2 \pm 3i\}$ which looks to be only 3 roots; however, our factorization was $(x - 1)^2(2 \pm 3i) = (x - 1)(x - 1)(2 + 3i)(2 - 3i)$, so by counting those factors separately, we indeed have 4 roots for the 4^{th} degree equation.

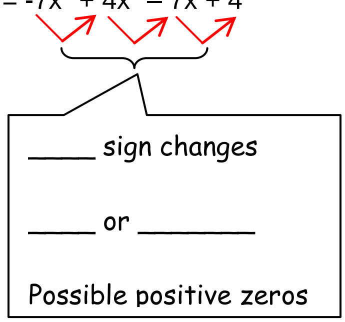
Using the above properties and the Linear Factorization Theorem (pg. 333), we can now find polynomials if given certain information.

Example: Find a 3^{rd} degree polynomial function f(x) with real coefficients such that 2 and 5i are zeros and $f(-1) = -234$ (write answer in standard form)

By Descartes' Rule of Signs (pg. 334), the number of positive and negative real roots possible within a polynomial function f(x) can be examined by looking at the sign changes within f(x) and f(-x)

Example: Determine the possible numbers of positive and negative real zeros of $f(x) = -7x^3 + 4x^2 - 7x + 4$

$f(x) = -7x^3 + 4x^2 - 7x + 4$ $\qquad\qquad$ $f(-x) =$



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