Section 2.6 (Rational Functions and Their Graphs)

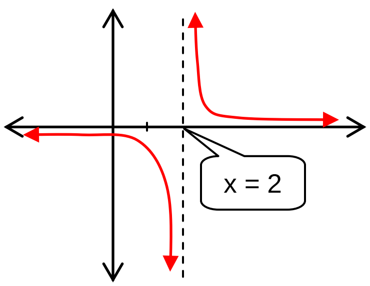
Rational functions are quotients of polynomial functions that can be expressed as

$f(x)=\frac{p(x)}{q(x)}$, where p and q are polynomial functions and $q(x) \neq 0$

Example: Find the domain of each rational function (all real #'s where the denominator does not equal 0)

$f(x)=\frac{x^2-25}{x-5}$ $g(x)=\frac{-2x}{x^2+6x-16}$ $h(x)=\frac{x+5}{x^2+25}$

{x | x ≠ ________}



Given the function $f(x)=\frac{1}{x-2}$, we see that x = 2 is not in the domain of f(x), but what happens as x approaches x = 2?

x approaches 2 from left (denoted as $x \to 2^-$), $f(x) \to$ _______

f(0) = -1/2 f(1) = -1 f(1.99) = -100 …

x approaches 2 from right (denoted as $x \to 2^+$), $f(x) \to$ _______

f(4) = f(3) = f(2.001) = …

What happens to the above graph and f(x) as $x \to \infty$? as $x \to -\infty$?

Look at HW example with time

Notice how the graph above approaches but never touches the line x = 2. A line x = a (such as this) is called a ***vertical asymptote*** when f(x) increases or decreases without bound as x approaches a.

Given ***simplified*** rational function $f(x)=\frac{p(x)}{q(x)}$, if a is a zero of q(x), then x = a is a vert. asymptote of f(x)

Example: Find the vertical asymptotes, if any, of the given rational function…

$f(x)=\frac{x}{x^2-1}$ $g(x)=\frac{x-2}{x^2-4}$ $h(x)=\frac{x+5}{x^2+25}$

Using the graph above again, we see that as x gets bigger and bigger, f(x) approaches _____. A line y = b (such as this) is called a ***horizontal asymptote*** if $f(x) \to b$ as x increases or decreases without bound.

Given a rational function where the degree of the numerator is n and the degree of the denominator is m…

- If n < m, the x-axis (or y = 0) is the horizontal asymptote of the graph
- If n = m, the line y = leading coefficient of the numerator / leading coefficient of the denominator is the horizontal asymptote of the graph
- If n > m, the graph has no horizontal asymptote

Book problems: 1,3,7,910,13,21,25,27,29,31,33,49,55,59

Example: Find the horizontal asymptote, if any, of the graphs of the following…

$f(x)=\dfrac{9x^2}{3x^2+1}$ $\qquad g(x)=\dfrac{9x}{3x^2+1}$ $\qquad h(x)=\dfrac{9x^3}{3x^2+1}$

Numerator deg. =

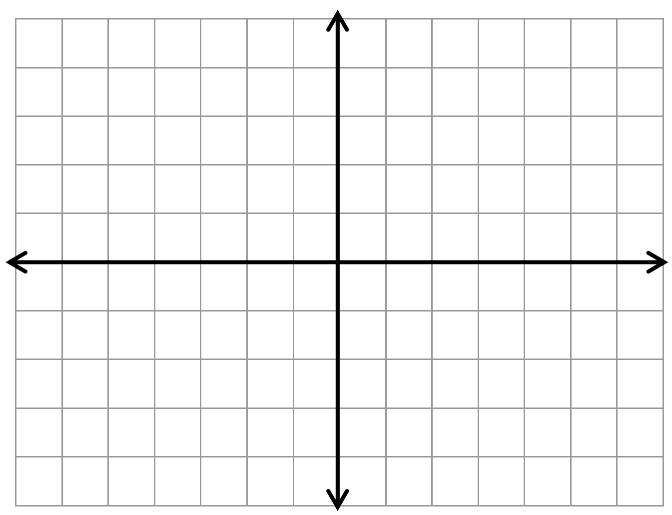
Denominator deg. =

The book lays out a seven step method for graphing rational functions (p. 347)

1. Determine symmetry => f(-x) = f(x) : y-axis symmetry, f(-x) = -f(x) : origin symmetry
2. Find y-intercept
3. Find x-intercept(s)
4. Find vertical asymptote(s)
5. Find horizontal asymptote(s)
6. Plot points between x-intercepts and vertical asymptotes (and other points if desired)
7. Graph the function

Examples: Use this method to graph the following…

$f(x)=\dfrac{3x-3}{x-2}$ $\qquad g(x)=\dfrac{x^2+x-2}{x^2-16}$



Book problems: 1,3,7,910,13,21,25,27,29,31,33,49,55,59