

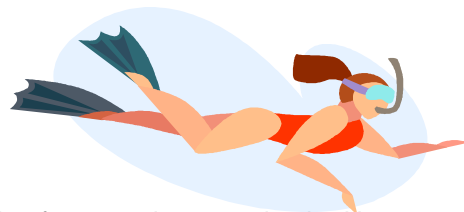
Section 2.8 (Modeling Using Variation)

When you swim underwater, the pressure in your ears depends on the depth at which you swim. The formula $p = 0.43d$ describes the water pressure, p , in pounds per square inch (psi), at a depth of d feet.

At a depth of 10 feet ($d = 10$), we have $p = 0.43(10) = 4.3$ psi

At a depth of 20 feet ($d = 20$), we have $p = 0.43(20) = 8.6$ psi

At a depth of 80 feet ($d = 80$), we have $p = 0.43(80) = 34.4$ psi

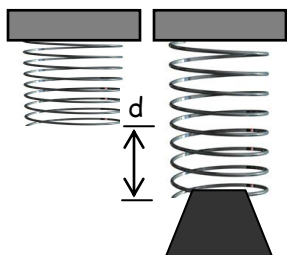


The formula $p = 0.43d$ illustrates that the water pressure is a constant multiple of your underwater depth. Here, the pressure p varies directly as the depth d (if d doubles, p doubles, etc.) – see book...

In functions, y **varies directly** as x (y is **directly proportional** to x) if there's a nonzero constant k such that $y = kx$ (constant k is called the constant of variation or the constant of proportionality) -- (note that these variations provide a linear relationship)

Examples: Suppose y varies directly with x . If y is 24 when x is 8, find y when x is 6.

Example: The number of gallons of water, W , used when taking a shower varies directly with time t . A shower lasting 5 min. used 30 gallons of water. How much water is used in a shower lasting 11 minutes?



Example: Hooke's Law – The distance a spring stretches is directly proportional to the weight attached to the spring. If a spring stretches 8 inches when a 56 lb. weight is attached to it, find the distance that a 35 lb. weight stretches the spring.

When y is proportional to the inverse of a variable x , we have $y = \frac{k}{x}$ and say that y **varies inversely** with x or y is **inversely proportional** to x

Examples: Suppose y varies inversely with x . If y is 6 when x is 3, find y when x is 9.

Example: The speed r at which one needs to drive in order to travel a constant distance is inversely proportional to the time t . If I can travel to Blacksburg to watch the Hokies whip Miami in 8 hours at 50 mph, how fast will I need to drive in order to make the trip in 5 hours?

Note that we can also say that a variable y varies directly with the square (or cube, etc.) of x as $y = kx^2$

Example: The distance, s , that a body falls from rest varies directly as the square of the time, t , of the fall. If skydivers fall 64 feet in 2 seconds, how far will they fall in 4 seconds?

In combined variation, direct and inverse variation can occur at the same time.

Example: If y varies directly as x and inversely as z , and $y = 16$ when $x = 32$ and $z = 2$, find y when $x = 27$ and $z = 3$.

In joint variation, a variable y can vary directly as the product of 2 or more variables.

Example: The area of a triangle varies jointly as its base and height. Express the area in terms of base b and height h . Then solve for b .

Example: Now y varies jointly as x and z . $y = 25$ when $x = 2$ and $z = 5$. Find y when $x = 8$ and $z = 12$.

Example: Body-mass index (BMI) varies directly as one's weight (in pounds) and inversely as the square of one's height (in inches). A person who weighs 180 pounds and is 5 feet (60 inches) tall has a BMI of 35.15. What is the BMI for a 170-pound person who is 5 feet 10 inches tall?



Example: Write an equation that expresses the following relationship: x varies directly as m and inversely as the difference between y and a . Then solve the equation for y .