Section 3.3 (Properties of Logarithms)

Because of the relationship between exponential and logarithmic forms, we find that the properties of exponents correspond to the properties of logarithms

- **Product Rule**: The logarithm of a product is the sum of the logarithms

$$\log_b (MN) = \log_b M + \log_b N$$

Examples: Use the product rule to ***expand*** each logarithmic expression

$\ln (4x) =$ $\log_6 (7 \bullet 11) =$ $\log (100x) =$

- **Quotient Rule**: The logarithm of a quotient is the difference of the logarithms

$$\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$$

Example: Use the quotient rule to expand each logarithmic expression

$\log\left(\frac{x}{2}\right) =$ $\log_8\left(\frac{23}{x}\right) =$ $\ln\left(\frac{e^5}{11}\right) =$

- **Power Rule**: The logarithm of a number with an exponent is the product of the exponent and the logarithm of that number

$$\log_b M^n = n \log_b M$$

Examples: Use the power rule to expand each logarithmic expression

$\ln x^2 =$ $\log_6 (3^9) =$ $\ln \sqrt[3]{x} =$ $\log (x + 4)^2 =$

Book problems: 3,7,15,21,23,25,27,31,37,53,59,67,69,71,75

You can use more than one of these properties in combination to expand logarithmic expressions

Examples: Use the logarithmic properties to expand each logarithmic expression as much as possible

$\log_b (x^4 \sqrt[3]{y}) =$

$\log_5 \left(\frac{\sqrt{x}}{25y^3} \right) =$

Thinking of these properties in reverse allows us to ***condense*** logarithmic expressions

Examples: Write the following as a single logarithm

$\log_4 2 + \log_4 32$

$\log (7x + 6) - \log x$

$2 \log (x - 3) - \log x$

$2 \ln x + \frac{1}{3} \ln (x + 5)$

We can use the change-of-base property to find logarithms in other bases (besides 10 and e)

- **Change-of-base Property**: The logarithm of M with base b is equal to the logarithm of M with any base divided by the logarithm of b with that same new base

$$\log_b M = \frac{\log_a M}{\log_a b} \qquad \text{(note that } \log_b M = \frac{\log M}{\log b} = \frac{\ln M}{\ln b}\text{)}$$

Example: Use common logarithms (base 10) to evaluate $\log_7 2506$

Example: Use natural logarithms to evaluate $\log_7 2506$

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