Name: _____

Thursday, December 13, 2012

MTH 113 Final Exam (practice - solutions)

1. (X pts.) Convert the following angles (1 of each problem):

   50^o from degrees to radians (leave π in your answer if applicable)

   50 * π/180 = 5π/18

   $-\frac{5\pi}{3}$ from radians to degrees

   –5π/3 * 180/π = -300^o

2. (X pts.) Find the reference angle **θ'** for each of the following angles (2 problems)

   θ = 240^o **θ' = 60^o**

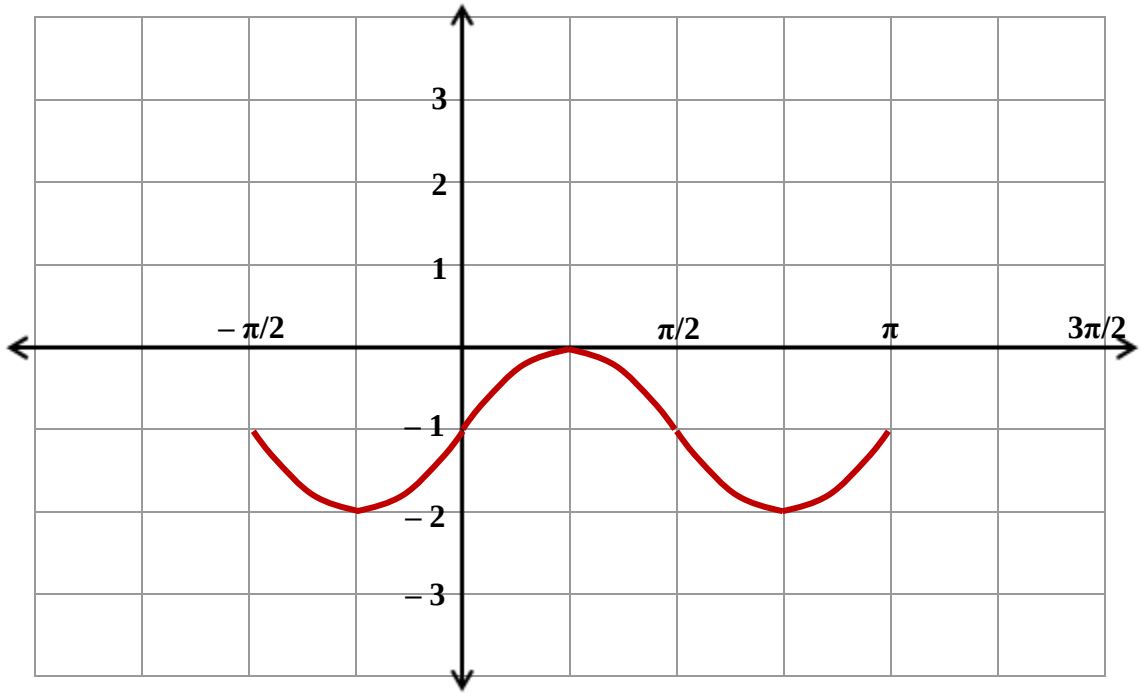
   θ = 5π/6 **θ' = π/6**

3. (X pts.) Find the exact value of the following functions (leave in any radical signs and simplify answer) (2)

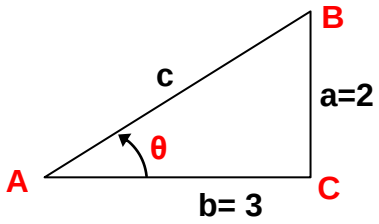
   cos $\left(\frac{\pi}{4}\right)$ **= $\sqrt{2}$ / 2**

   sin (210^o) **= – 1 / 2**

4. (X pts.) Sketch the graph of **y = sin (2x) – 1** over the interval $-\frac{\pi}{2} \leq x \leq \pi$ (1)



5. (X pts.) Find the following for the given right triangle (simplify answer) (2)



$c^2 = 4 + 9 \qquad c = \sqrt{13}$

sin θ = $2 / \sqrt{13} = 2\sqrt{13} / 13$

cos θ = $3 / \sqrt{13} \qquad = 3\sqrt{13} / 13$

tan θ = $2 / 3$

6. (X pts.) The flagpole in the middle of Sesame Street is 12 meters tall and casts a shadow 8 meters long. Sketch the problem and find the angle of elevation of the sun to the nearest tenth of a degree. (1)

$\tan \theta = 12 / 8 \qquad \tan^{-1} (3/2) = \sim 56.3^{o}$

7. (X pts.) Verify the following trigonometric identities… (3)

$\sin x - \sin x \cos^2 x = \sin^3 x$

$\sin x - \sin x(1 - \sin^2 x)$
$\sin x - \sin x + \sin^3 x$
$\sin^3 x$

$\cot(\theta) \cos(\theta) + \sin(\theta) = \csc(\theta)$

$[\cos x / \sin x]\cos x + \sin x$
$\cos^2 x / \sin x + \sin x$
$(\cos^2 x + \sin^2 x) / \sin x$
$1 / \sin x = \csc x$

$$\frac{\cos(\alpha-\beta)}{\cos\alpha\cos\beta} = 1 + \tan(\alpha)\tan(\beta)$$

$(\cos a \cos b) / (\cos a \cos b) + (\sin a \sin b) / \cos a \cos b = 1 + \tan a \tan b$

$$\tan\theta = \frac{\sin 2\theta}{1+\cos 2\theta}$$

$(2 \sin a \cos a) / (1 + (2\cos^2 a - 1))$
$(2 \sin a \cos a) / (2\cos^2 a)$
$[2/2] [\sin a / \cos a] [\cos a / \cos a]$
$\tan a$

8. (X pts.) Solve each equation on the interval [0, 2π) (2)

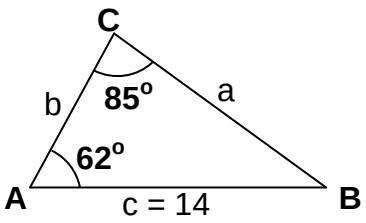
$\tan 2x = \sqrt{3}$

$2x = \pi/3$
$x = \pi/6 + n\pi$
$x = \{\pi/6, 7\pi/6\}$

$4 \cos^2 x - 3 = 0$

$\cos^2 x = 3/4$
$\cos x = +/- \sqrt{3} / 2 + 2n\pi$
$x = \{\pi/6, 5\pi/6, 7\pi/6, 11\pi/6\}$

9. (X pts.) Solve the following oblique triangles (round to nearest tenth)… (1/4)



SAA and/or ASA case (Law Of Sines)…

B = 180 – (62 + 85) = 180 – 147 = 33.0°

b / sin 33° = 14 / sin 85° => sin 33° (14/sin 85°) => b = 7.7

a / sin 62° = 14 / sin 85° => sin 62° (14/sin 85°) => a = 12.4

10. (X pts.) Due to a mistake in flight plans, two airplanes (one with the players, the other with the equipment) leave Tuscaloosa headed for Alablehma's bowl game at the exact same time. One flies a straight path on a bearing of **S35°E** (towards the Outback Bowl) at **280 miles per hour**. The other plane flies a straight path on a bearing of **S82°W** (towards the Cotton Bowl) at **285 miles per hour**. How far apart will the airplanes be after 2 hours? (1)

plane 1 dist. = 280*2 = 560 miles plane 2 dist. = 285*2 = 570 miles
Angle btwn flights = 117°

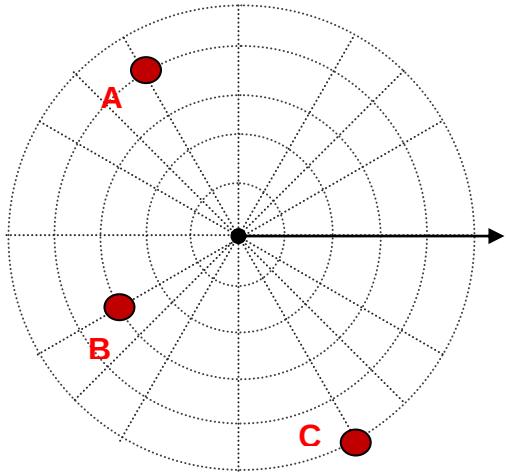
$c^2 = 560^2 + 570^2 - 2(560)(570)\cos(117^\circ)$
$c^2 = 638500 - 638400\cos(117^\circ)$
c = 963.5 miles

11. (X pts.) Plot and label the following polar coordinates (r,θ)…(2)

A: $\left(4, \frac{2\pi}{3}\right)$

B: $\left(-3, \frac{\pi}{6}\right)$

C: $\left(5, \frac{-7\pi}{3}\right)$



12. (X pts.) Find the rectangular coordinates of the point whose polar coordinates are $\left(3, \frac{5\pi}{6}\right)$ (1)

x = 3cos(5π/6) = 3(-sqrt(3)/2) = -3sqrt(3)/2

y = 3sin(5π/6) = 3(1/2) = 3/2

(-3sqrt(3)/2, 3/2)

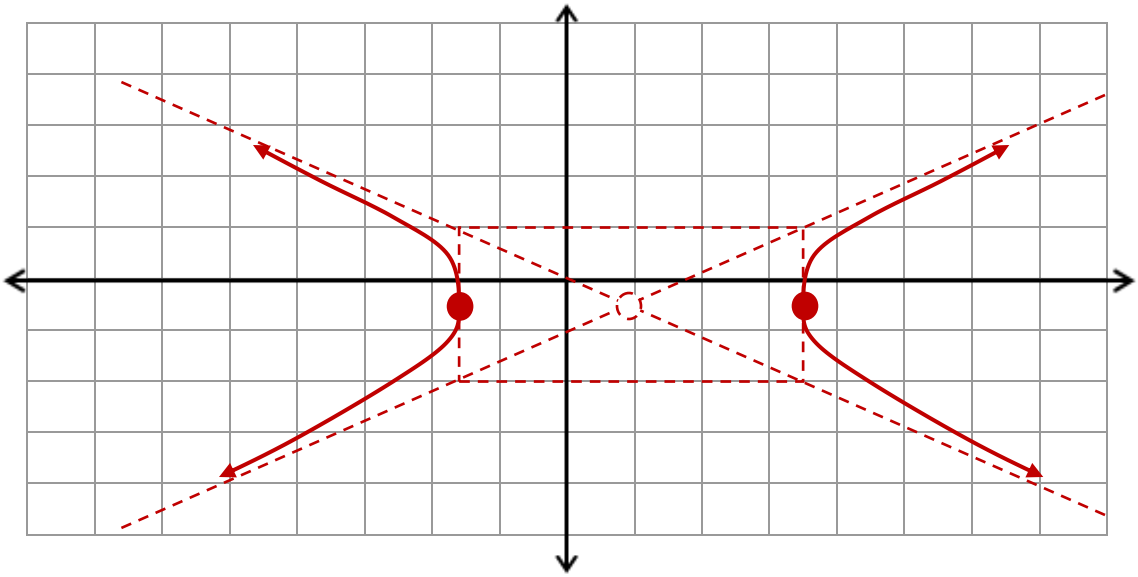
13. (X pts.) If **v** = 3**i** + 2**j** and **w** = 5**i** – 3**j**, find each of the following vectors… (3)

v – w	3**v** + 4**w**	**v • w**
v – w = –2i + 5j	3v+4w = (9i+6j) + (20i–12j) = 29i – 6j	= 15 – 6 = 9

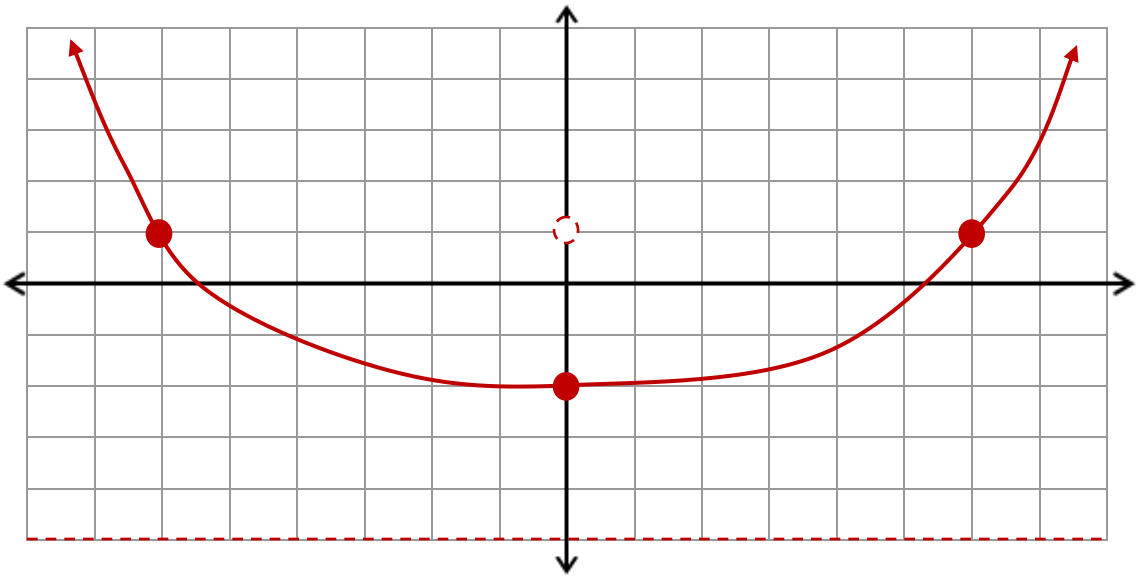
14. (X pts.) The jet stream is blowing at **60 miles per hour** in the direction of **N 45° E**. Express its velocity as a vector (1)

v = 60cos(45°) i + 60sin(45°) j
v = 30sqrt(2) i + 20sqrt(2) j

15. (X pts.) Sketch the graph of $\frac{(x-2)^2}{25} - \frac{(y+1)^2}{9} = 1$ (1)



16. (X pts.) Find focus and directrix of the parabola given and graph $x^2 = 12(y + 2)$ (1)



Focus => 4p=12 => p=3 => (0,1)

Directrix => y = – 5

17. (X pts.) Find the standard form of an ellipse with foci at (0,– 6) and (0,6) and vertices (0,– 10) and (0,10) (1)

- Or multiple choice – choose the correct graph / choose the correct equation -

a = 10, c = 6

$36 = 100 - b^2 \Rightarrow b = 8$

$$\frac{x^2}{100} + \frac{y^2}{64} = 1$$

18. (X pts.) Convert the equation into standard form $4x^2 + y^2 - 8x + 4y - 8 = 0$ (1 complete the square)

$(4x^2 - 8x) + (y^2 + 4y) = 8$
$4(x^2 - 2x + 1) + (y^2 + 4y + 4) = 8 + 4 + 4$

$4(x - 1)^2 + (y + 2)^2 = 16$

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$$

19. (X pts.) Identify the type of graph given by $3x^2 + 2y^2 + 12x - 4y + 2 = 0$ (1)

AC = 3(2) > 0, ellipse

20. (X pts.) Cookie Monster is making an effort to go green, so he has decided to make a solar cooker to help him bake cookies (satellite dish basically) with a diameter of 6 feet and a depth of 1 foot. Where should he put the cooker (focus) to get the maximum reflected sun rays to bake the cookies? (1)

p = ? $x^2 = 4py \Rightarrow 3^2 = 4p(1) \Rightarrow p = 9/4 \Rightarrow$ put cooker 9/4 feet from bottom

21. (X pts.) A trigonometry book is launched by an angry student with an initial velocity of 200 ft. per second at an angle of 42^o with the horizontal. The book was launched from a height of 4 feet. (2)

a) Find the parametric equations that describe the position of the book as a function of time. (1)

$x = (200 \cos 42^o)t$
$y = 4 + (200 \sin 42^o)t - 16t^2$

b) Describe the book's position after t = 1 second and after t = 3 seconds (1)

t = 1 => 148.6 ft. away horizontally, 121.8 ft. in the air
t = 3 => $600 \cos 42^o = 445.9$ ft. away horizontally, $4 + (600 \sin 42^o) - 144 = 261.5$ ft. in the air

BONUS QUESTIONS ???