Name: ____________________________

Thursday, December 12, 2013

MTH 113 Final Exam

1. (6 pts.) Find the direction, vertex, focus, and directrix of the parabola given by $\mathbf{x^2 = -12(y - 3)}$

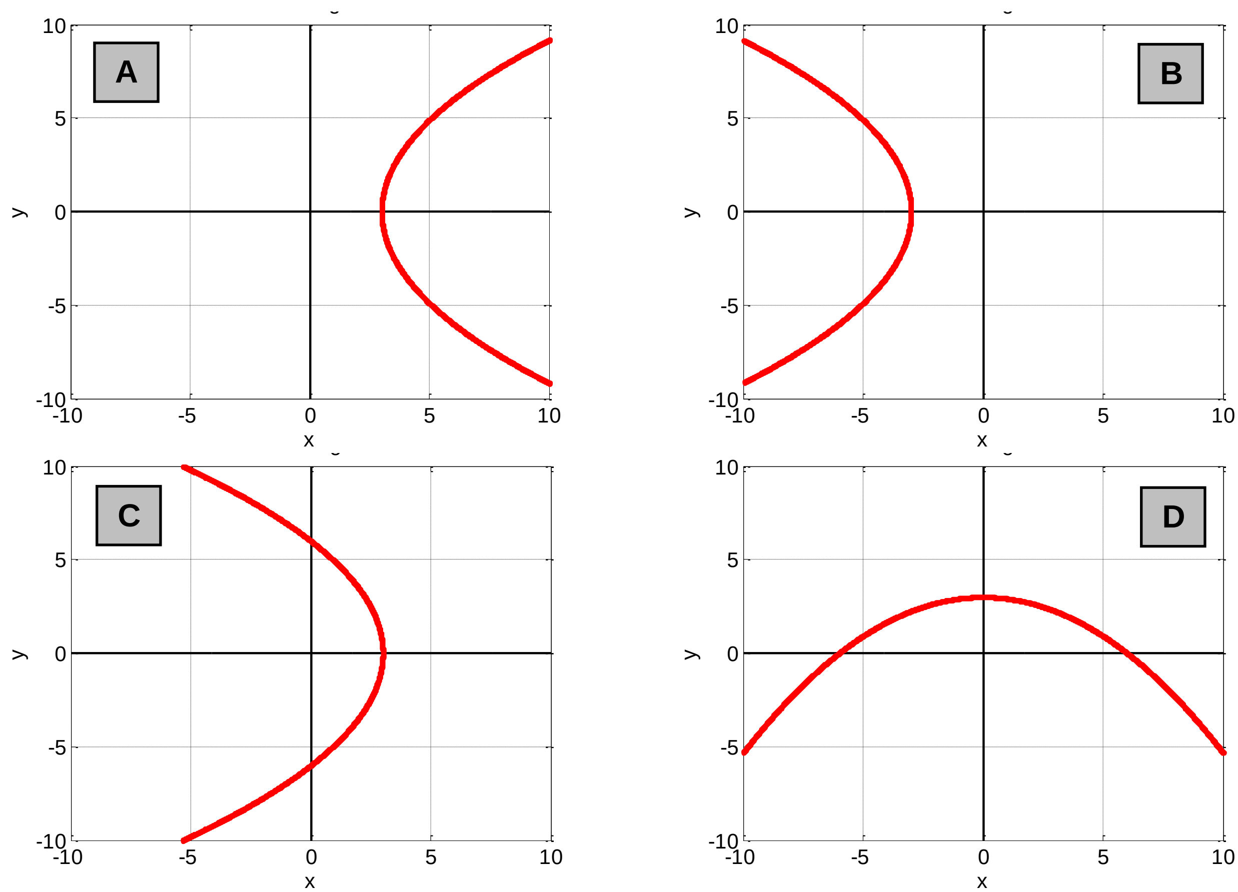
Opens (up/down/left/right):

Vertex (point):

Focus (point):

Directrix (line):

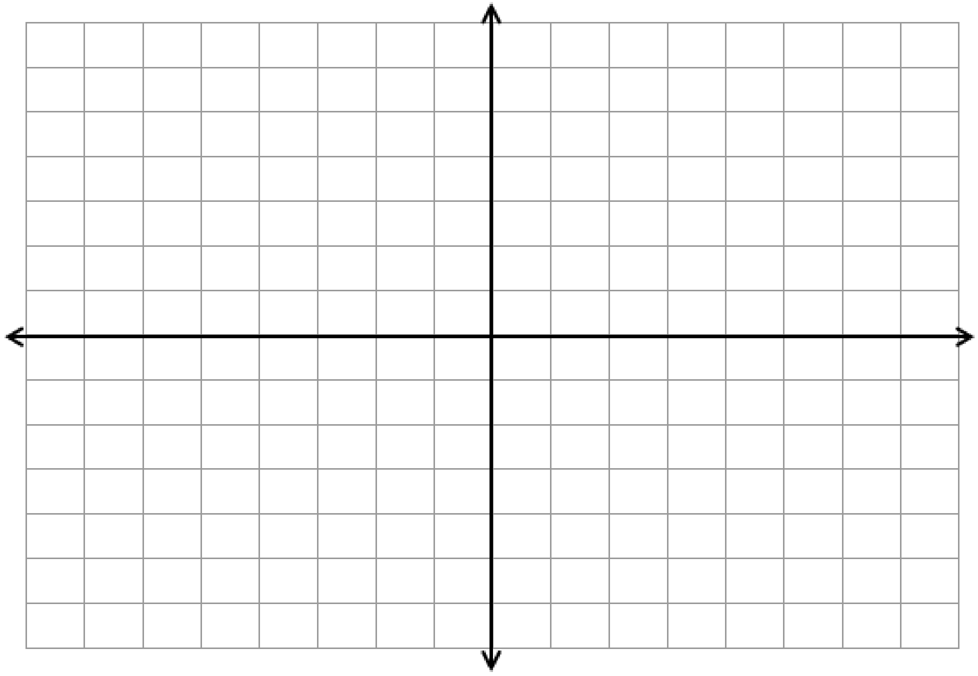
2. (3 pts.) Which figure represents the equation in previous question? _______________



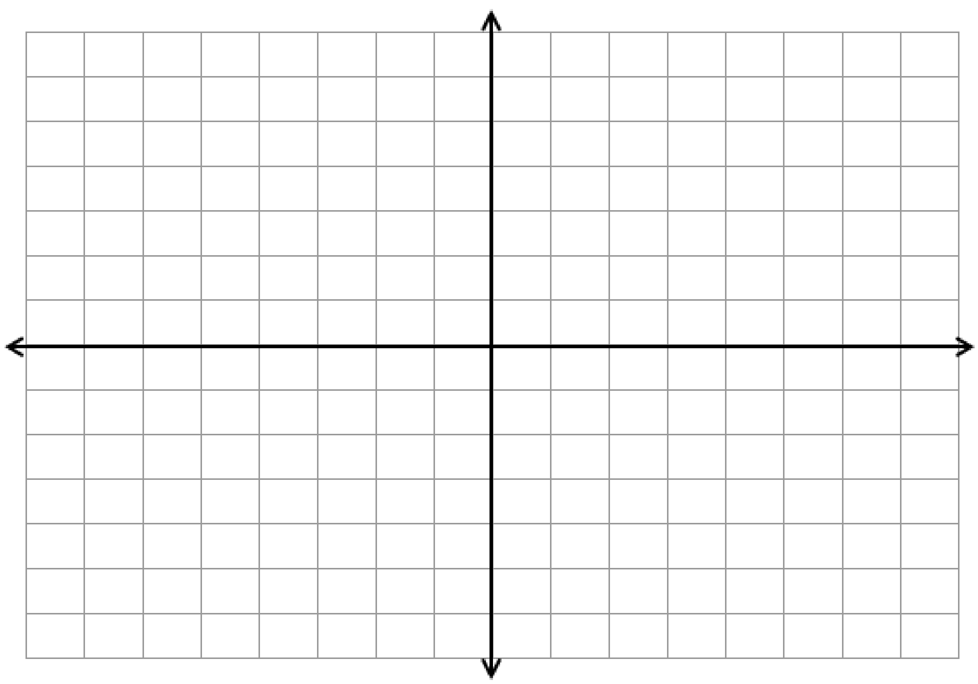
3. (4 pts.) Complete the square to convert the equation $\mathbf{7x^2 - 3y^2 + 42x - 12y + 4y - 8 = 0}$ into standard form <u>and</u> determine the type of conic section represented

Type of graph?

4. (5 pts.) Sketch the graph of $\frac{(x-2)^2}{4} + \frac{(y+1)^2}{16} = 1$



5. (4 pts.) Graph the plane curve defined by the parametric equations $x = t^2$, $y = t - 2$, $-2 \le t \le 2$



6. (5 pts.) Cookie Monster is making an effort to go green, so he has decided to make a solar cooker to help him bake cookies (satellite dish basically) with a diameter of **8 feet** and a depth of **2 feet**. At what height should he put the cooking platform (focus) to get the maximum reflected sun rays to bake the cookies?

7. (6 pts.) Convert the following angles (simplify by leaving π in answers and reducing fractions):

80^o from degrees to radians

$\frac{11\pi}{10}$ from radians to degrees

8. (6 pts.) Find the reference angle **θ'** for each of the following angles (angle measure should be between 0-360 degrees or 0 and 2π radians – leave π in the answer when converting to radians)

$\theta = 135^o$

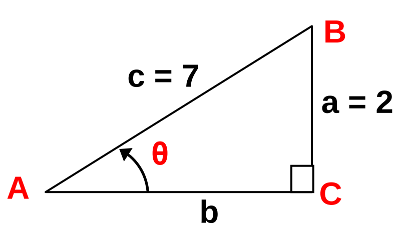
$\theta = -\frac{5\pi}{6}$

9. (6 pts.) Find the exact value of the following functions (leave in any radical signs and simplify answer, leaving no radical signs in the denominator)

$\cos\left(\frac{4\pi}{3}\right)$ =

$\tan(315^o)$ =

10. (6 pts.) Find the following for the given right triangle (simplify answer while leaving in radical signs)



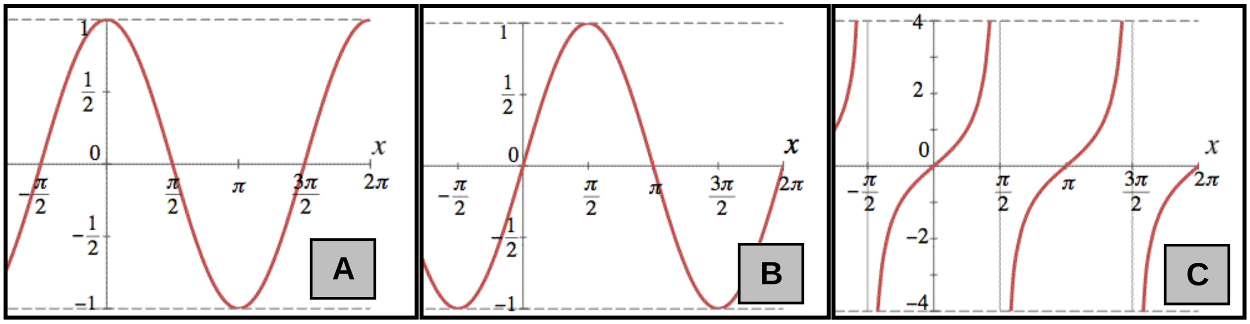
sin θ =

cos θ =

tan θ =

11. (5 pts.) Match the correct graph to the given function…

sin x = _________ , cos x = _________ , tan x = _________



12. (5 pts.) In appreciation of your awesome trigonometry teacher and out of respect to the awesomeness of the Hokies you have decided to hoist a Virginia Tech flag in your front yard. You placed a 12 foot pole in the yard, and at one point during the day, it casted a 7 foot shadow. At what <u>angle of elevation</u> was the sun from the point on the ground where you measured the shadow (leave 2 decimal places)?

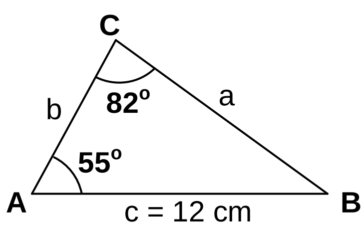
13. (6 pts.) Verify the following trigonometric identities

sin(x) cos(x) tan(x) = 1 – cos²(x) $\qquad$ $\frac{2\cos(2\theta)}{\sin(2\theta)}$ **= cot(θ) – tan(θ)**

14. (6 pts.) Solve each equation and list all solutions on the interval $[0, 2\pi)$

$5 \sin(x) = 3 \sin(x) + 1$ **$\cos^2(x) - 3\cos(x) + 2 = 0$**

15. (6 pts.) Solve the following oblique triangle (round to 2 decimal places)

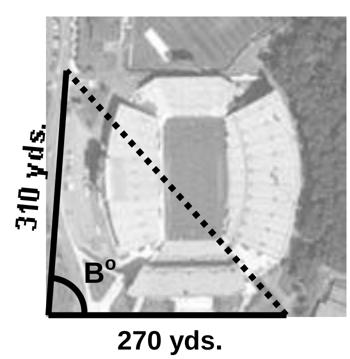


B =

b =

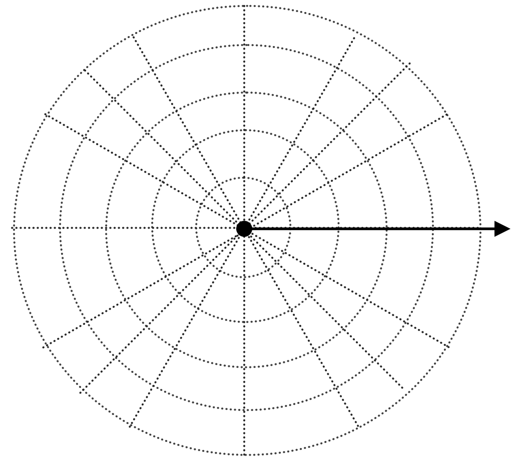
a =

16. (5 pts.) Imagine you and a friend have decided to walk around beautiful Lane Stadium at Virginia Tech. Starting from a point outside the stadium, you walk directly to the east while your friend walks on a bearing of **N18°E**. After 10 minutes, you covered a distance of **270 yards**, and your friend travelled **310 yards**. Calculate the distance between you and your friend at this point (2 decimal places).



17. (3 pts.) Plot and label the following polar coordinates (r,θ)

$$\left(-3, \frac{5\pi}{6}\right)$$



18. (4 pts.) Find the rectangular coordinates of the point whose polar coordinates are $\left(4, \frac{2\pi}{3}\right)$ -- leave in radical signs as needed

19. (6 pts.) If **v** = 7**i** + 6**j** and **w** = 2**i** – 7**j**, find each of the following vectors

v – w 3**v** + 4**w** **v • w**

Possibly Useful Formulas (let me know if I missed any)

$\theta = s/r$ $v = s / t$ $y = A \sin(Bx - C) + D$ amplitude = $|A|$ period = $2\pi/B$

Reciprocal

$\sin(t) = \frac{1}{\csc(t)}$ $\csc(t) = \frac{1}{\sin(t)}$

$\cos(t) = \frac{1}{\sec(t)}$ $\sec(t) = \frac{1}{\cos(t)}$

$\tan(t) = \frac{1}{\cot(t)}$ $\cot(t) = \frac{1}{\tan(t)}$

Quotient

$\tan(t) = \frac{\sin(t)}{\cos(t)}$

$\cot(t) = \frac{\cos(t)}{\sin(t)}$

Pythagorean

$\sin^2 t + \cos^2 t = 1$

$\sin^2 t = 1 - \cos^2 t$

$\cos^2 t = 1 - \sin^2 t$

$1 + \tan^2 t = \sec^2 t$

$1 + \cot^2 t = \csc^2 t$

Even-Odd

$\sin(-x) = -\sin(x)$

$\cos(-x) = \cos(x)$

$\tan(-x) = -\tan(x)$

$\csc(-x) = -\csc(x)$

$\sec(-x) = \sec(x)$

$\cot(-x) = -\cot(x)$

Sum-to-Product Formulas

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

Double-Angle Formulas

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ --- or --- $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ $a^2 = b^2 + c^2 - 2(bc)\cos A$ …

$x^2 + y^2 = r^2$ $x = r\cos\theta$ $y = r\sin\theta$ $\tan\theta = y / x$

$||\mathbf{v}|| = \sqrt{a^2 + b^2}$ $\mathbf{u} = \frac{\mathbf{v}}{||\mathbf{v}||}$ $\mathbf{v} = ||\mathbf{v}|| \cos\theta\, \mathbf{i} + ||\mathbf{v}|| \sin\theta\, \mathbf{j}$

$\mathbf{v} \bullet \mathbf{w} = ac + db$ $\mathbf{v} \bullet \mathbf{w} = ||\mathbf{v}||\,||\mathbf{w}||\cos\theta$ $\cos\theta = \mathbf{v} \bullet \mathbf{w} / ||\mathbf{v}||\,||\mathbf{w}||$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (or a and b reversed)

$c^2 = a^2 - b^2$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ (or a and b reversed)

$c^2 = a^2 + b^2$

$(x - h)^2 = 4p(y - k)$ or $(y - k)^2 = 4p(x - h)$