Name: ______________________________

Thursday, March 13, 2014

MTH 113 Test 2 (Chapter 5)

1. (14 pts.) Suppose that **sin α =** $\frac{4}{5}$ for a quadrant II angle α and **sin β =** $\frac{5}{13}$ for a quadrant I angle β. Find the exact value of the following (hint: find sin / cos / tan of the angles first – simplify / leave in radical signs)…

sin (α – β)

cos α = -3/5, cos β = 12/13

= sin α cos β – cos α sin β
= (4/5)(12/13) – (-3/5)(5/13)
= 48/65 + 15/65
= 63/65

cos (α + β)

= cos α cos β – sin α sin β
= (-3/5)(12/13) – (4/5)(5/13)
= -36/65 – 20/65
= -56/65

2. (6 pts.) Find the exact value (include radical signs / leave no radical sign in the denominator as necessary) of **tan(105^o) = tan(45^o + 60^o)** using the sum formula

tan 45 = 1, tan 60 = sqrt(3)

tan(45+60) = [tan(45) + tan(60)] / [1 – tan(45)tan(60)]
= [1 + sqrt(3)] / [1 – 1sqrt(3)]
= [1 + sqrt(3)] / [1 – 1sqrt(3)] * [1 + sqrt(3)] / [1 + sqrt(3)]
= [1 + 2sqrt(3) + 3] / [1 – 3]
= [4 + 2sqrt(3)] / -2
= -2 – sqrt(3)

3. (14 pts.) Suppose that **sin θ =** $\frac{7}{9}$ for a quadrant I angle θ. Find the exact value of the following (leave in radical signs / simplify)…

sin (2θ)

cos (2θ)

a = sqrt(81-49) = sqrt(32) =4sqrt(2), cos(θ) = 4sqrt(2)/9

= 2sin(θ)cos(θ)
= 2(7/9)(4sqrt(2)/9)
= 56sqrt(2)/81

= $\cos^2$(θ) – $\sin^2$(θ)
= 16(2)/81 – 49/81
= 32/81 – 49/81
= -17/81

4. (8 pts.) Use a half-angle formula to find the exact value (leave in radical signs as necessary) of **tan (75°)**

--- hint: use one of the tan half-angle formulas without a radical sign ---

tan(75) = tan(150/2) = [1 – cos(150)] / sin(150)
= [1 – -sqrt(3)/2] / [1/2]
= [(2 + sqrt(3))/2] * [2]
= 2 + sqrt(3)

5. (10 pts.) Express each sum or difference as a product (if possible, find this product's exact value)

cos(5x) + cos(3x)

= 2cos(8x/2)cos(2x/2)
= 2cos(4x)cos(x)

sin(165°) – sin (105°)

= 2sin(60/2)cos(270/2)
= 2sin(30)cos(135)
=2(1/2)(-sqrt(2)/2)
= -sqrt(2)/2

6. (10 pts.) Express each product as a sum or difference

sin(7x) cos(3x)

½[sin(10x) + sin(4x)]

sin(5x) sin(2x)

½[cos(3x) – cos(7x)]

7. (16 pts.) Solve each equation (find all solutions on the interval [0, 2π) answer in radians)

$$\cos 2x = \frac{1}{2}$$

2x = π/3 + 2nπ
x = π/6 + nπ
x = π/6, 7π/6

2x = 5π/3 + 2nπ
x = 5π/6 + nπ
x = 5π/6, 11π/6

x = { π/6, 5π/6, 7π/6, 11π/6}

$$\sin^2(\theta) - 5 = 4\sin(\theta)$$

$\sin^2(\theta) - 4\sin(\theta) - 5 = 0$
(sin(θ) – 5)(sin(θ) + 1) = 0
sin(θ) = 5 --- NOT POSSIBLE
sin(θ) = -1
θ = 3π/2

8. (7 pts.) With Easter approaching, the Easter Bunny is practicing his egg-throwing skills. If he wants to throw his eggs with an initial velocity of $v_0 = 90$ feet per second, at what angle of elevation will he need to throw it to intercept someone standing 100 feet away (round answer to 2 decimal places)? Use the formula…

$$d = \frac{{v_0}^2}{16} \sin\theta \cos\theta$$

$d = (v^2/32)(2\sin(\theta)\cos(\theta))$
$d = (v^2/32)(\sin(2\theta))$
$100 = (8100/32)(\sin(2\theta))$
$3200 = 8100\sin(2\theta)$
$0.395 = \sin(2\theta)$
$2\theta = 23.266^o$
$\theta = 11.633^o$

BONUS: