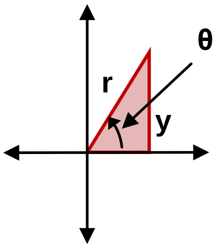
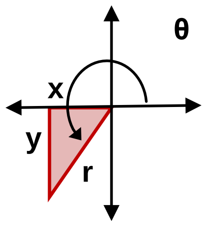
Section 4.4 (Trigonometric Functions of Any Angle)

In the last section, we focused on the trigonometric functions of acute angles located in the first quadrant by forming right triangles and drawing line segments perpendicular to the x-axis. We can extend this concept to other angles, using similar techniques.





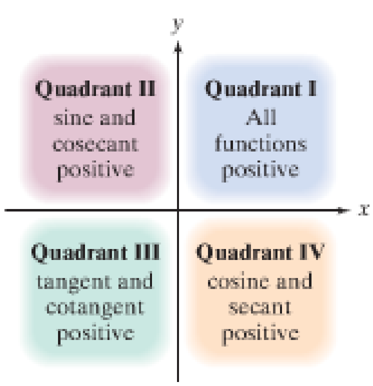
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenouse}} = \dfrac{y}{r}$	*$\csc\theta = \dfrac{\text{hypotenouse}}{\text{opposite}} = \dfrac{r}{y}$
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenouse}} = \dfrac{x}{r}$	*$\sec\theta = \dfrac{\text{hypotenouse}}{\text{adjacent}} = \dfrac{r}{x}$
*$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{y}{x}$	*$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{x}{y}$

* → Note that these functions are undefined when denominator is 0

Example: Let **P = (1, – 3)** be a point on the terminal side of **θ**. Find each of the 6 trigonometric functions of **θ**.

Example: Evaluate, if possible, the sine, cosine, and tangent of the following

$\theta = 0^o = 0$ **$\theta = 90^o = \pi/2$** **$\theta = 180^o = \pi$** **$\theta = 270^o = 3\pi/2$**



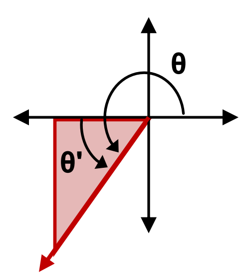
By examining the definition of our trigonometric functions, one can determine if the sign of a trig. function based on which quadrant the angle lies (see discussion on book pg. 504 for more details.

Example: Sketch a random angle in quadrant II on the board to determine the signs of the trig. functions.

Example: If **$\sin\theta < 0$** and **$\cos\theta < 0$**, name the quadrant where the angle lies.

Example: Given **$\tan\theta = -1/3$** and **$\cos\theta < 0$**, find **$\sin\theta$** and **$\sec\theta$**

The process described to process right triangles by dropping a line perpendicular to the x-axis results in the use of an acute angle θ' formed by the terminal side of θ and the x-axis. This angle formed between the terminal side and the x-axis is called a reference angle.



Book problems:

Example: Find the reference angle **θ'** for each of the following angles

θ = 210° **θ = 7π/4** **θ = –240°** **θ = 3.6**

If the given angle θ is > 360° or < – 360°, we start by finding the coterminal angle before getting the reference angle

Example: Find the reference angle for each of the following angles

θ = 665° **θ = 15π/4** **θ = –11π/3**

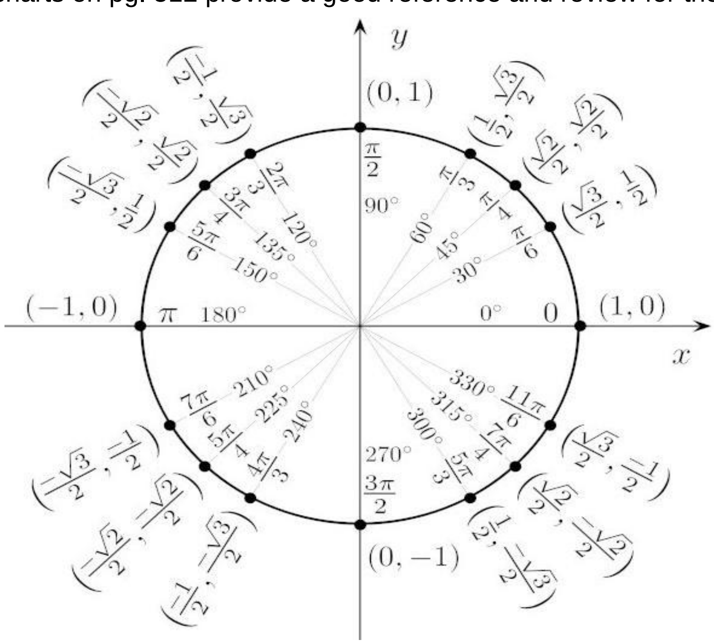
We can utilize these reference angles to evaluate trigonometric functions. The book describes a 2-step process for these evaluations by (1) finding the associated reference angle and its value and (2) then applying the appropriate sign based on the quadrant in which the angle lies (see pg. 508 and examples). I prefer a similar method while sketching out the appropriate triangle and signs to determine the trigonometric functions.

Example: Use reference angles to find the exact value of the following trigonometric functions

sin 300° **tan 5π/4** **sec (–π/6)**

sin 17π/6 **sin (–22π/3)**

The tables and charts on pg. 512 provide a good reference and review for the material covered



Book problems: