Section 5.2 (Sum and Difference Formulas)

Cosine of the difference of two angles (see pg. 597 and below steps for visual proof or explore on board with time)

- Start with unit circle and angles α and β having endpoints P and Q
- List coordinates of P and Q for these angles (using cos α for x and sin α for y)
- Connect P and Q with line segment and apply distance formula
- Rotate triangle in line with x-axis and maintaining angle α – β
- Re-apply distance formula to get equivalent distance between P and Q (one point should now be (1,0))

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Example: Find the exact value of cos 30^o using only 90^o and 60^o angles

Example: Find the exact value of cos 80^o cos 20^o + sin 80^o sin 20^o

Example: Verify the identity $\frac{\cos(\alpha-\beta)}{\cos\alpha\cos\beta} = 1 + \tan\alpha\tan\beta$

We can use this difference formula to verify and establish other identities (see cofunction complements discussion pg. 599 that shows $\cos(\pi/2 - \theta) = \sin\theta$)

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

Example: Suppose that sin α = 4/5 for a quadrant II angle α and sin β = ½ for a quadrant I angle β. Find the exact value of the following…

cos α cos β cos (α + β) sin (α + β)

Book problems:

If we rewrite tan (α + β) using sin and cos, we can develop a formula for the tangent of sums and differences

Example: Derive the identity for tan (α + β) -- Hint: divide top and bottom by cos α cos β

(Optional) Example: Derive the identity for tan (α – β) -- Hint: use tan (α – β) = tan [α +(– β)] (and tan is odd)

(Optional) Example: Verify the identity tan (x + π) = tan x

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Example (Extra Credit?): (#80 – pg. 606) A tuning fork is held a certain distance from your ears and struck. Your eardrums' vibrations after t seconds are given by p = 3 sin (2t). When a second tuning fork is struck, the formula p = 2 sin (2t + π) describes the effects of the sound on the eardrums' vibrations. The total vibrations then are given by p = 3 sin (2t) + 2 sin (2t + π).

- Simplify p to s single term containing the sin

- If the amplitude of p is zero, no sound is heard. Based on your equation in the part above, does this occur with the two tuning forks in the exercise (Explain)

Book problems: