Section 5.4 (Product-to-Sum and Sum-to-Product Formulas)

We can write products of sin and / or cos functions as sums or differences…

Product-to-Sum Formulas

$$\sin \alpha \sin \beta = \tfrac{1}{2} [\cos (\alpha - \beta) - \cos (\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \tfrac{1}{2} [\cos (\alpha - \beta) + \cos (\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \tfrac{1}{2} [\sin (\alpha + \beta) + \sin (\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \tfrac{1}{2} [\sin (\alpha + \beta) - \sin (\alpha - \beta)]$$

We can derive these formulas using the difference and sum formulas of cos (and sin) – we'll look at the 2nd formula.

Add difference and sum formulas for cos =>

$$\begin{array}{rl} & \cos (\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ + & \underline{\cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta} \\ & \cos (\alpha - \beta) + \cos (\alpha + \beta) = 2 \cos \alpha \cos \beta \ \ldots \end{array}$$

Example: Express each of the following products as a sum or difference…

$\sin 5x \sin 2x$ $\qquad$ $\cos 7x \cos x$

Sum-to-Product Formulas

$$\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2}\right) \cos \left(\frac{\alpha - \beta}{2}\right)$$

$$\sin \alpha - \sin \beta = 2 \sin \left(\frac{\alpha - \beta}{2}\right) \cos \left(\frac{\alpha + \beta}{2}\right)$$

$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha + \beta}{2}\right) \cos \left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha + \beta}{2}\right) \sin \left(\frac{\alpha - \beta}{2}\right)$$

We can derive these formulas using the Product-to-Sum Formulas above and going in reverse (see pg. 621)

Consider the right side from the 2nd formula above $\qquad 2 \sin \left(\frac{\alpha - \beta}{2}\right) \cos \left(\frac{\alpha + \beta}{2}\right)$

-- apply product-to-sum $\qquad 2 * \tfrac{1}{2} [\sin \left(\frac{\alpha - \beta}{2} + \frac{\alpha + \beta}{2}\right) + \sin \left(\frac{\alpha - \beta}{2} - \frac{\alpha + \beta}{2}\right)]$

-- simplify…

Book problems:

Example: Express each sum as a product…

sin 7x + sin 3x

cos 3x + cos 2x

Applying these identities in the numerator and denominator is often helpful in verifying identities that contain a fraction on one side with sums and differences of sin and/or cos

(Optional) Example: Verify the identity $\frac{\cos 3x - \cos x}{\sin 3x + \sin x} = -\tan x$

Review online HW and Examine #45 on pg. 624 (as time permits)

Book problems: