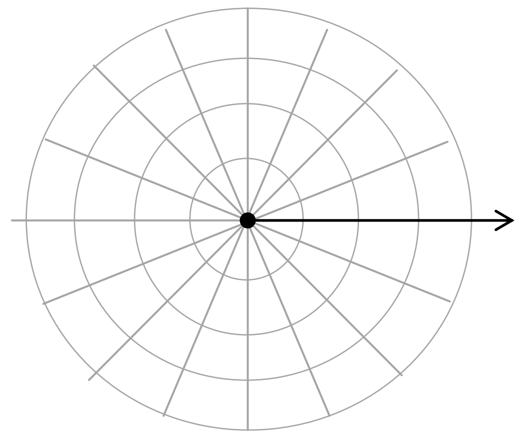
Section 6.4 (Graphs of Polar Equations – OPTIONAL – No HW)

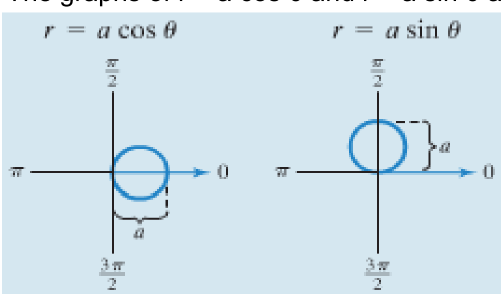
When plotting polar equations, we will typically utilize a polar grid. Just as with rectangular equations, we can use the point-plotting method to plot any polar equation

Example: See example 1 (pg. 675-6) and graph the equation r = 4 sin θ with θ in radians.

θ	r = 4 sin θ
0	
π/6	
π/3	
π/2	
2π/3	
5π/6	
π	



The graphs of r = a cos θ and r = a sin θ are circles (what produces a graph of a circle centered around the origin?)

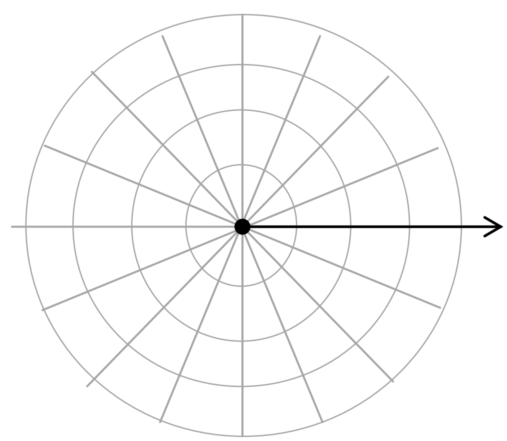


We can also use symmetry to graph polar equations more quickly

1. Replace θ with – θ: same eqn. => symmetric w/ polar (x) axis
2. Replace (r,θ) with (– r, – θ): same => symmetric w/ θ = π/2 (y)
3. Replace r with – r: same => symmetric w/ pole (origin)

Example: Check for symmetry and graph the polar eqn. r = 1 + cos θ

Polar Axis (x) Line θ = π/2 (y) Pole (origin)



Below is a summary of some other types of graphs of polar equations (review in the book)…

Limaçons

The graphs of

$$r = a + b\sin\theta, \quad r = a - b\sin\theta,$$
$$r = a + b\cos\theta, \quad r = a - b\cos\theta, \quad a > 0, b > 0$$

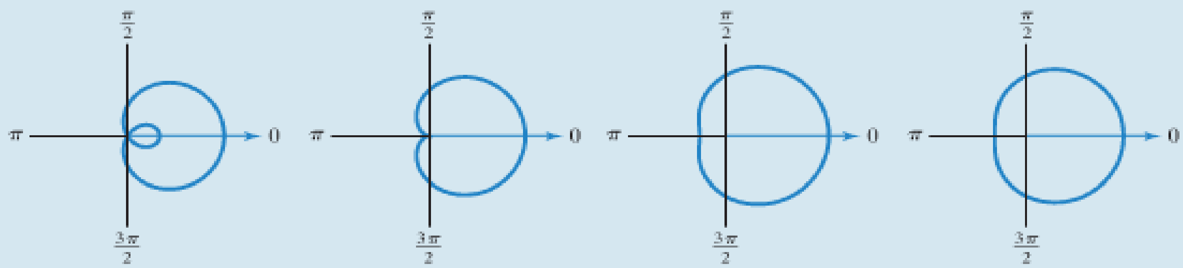
are called **limaçons**. The ratio $\frac{a}{b}$ determines a limaçon's shape.

Inner loop if $\frac{a}{b} < 1$

Heart-shaped if $\frac{a}{b} = 1$ and called cardioids

Dimpled with no inner loop if $1 < \frac{a}{b} < 2$

No dimple and no inner loop if $\frac{a}{b} \geq 2$



Rose Curves

The graphs of

$$r = a\sin n\theta \quad \text{and} \quad r = a\cos n\theta, \quad a \neq 0,$$

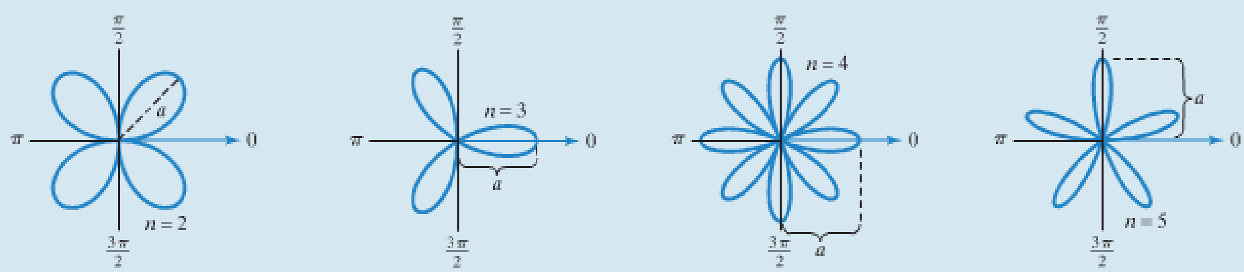
are called **rose curves**. If n is even, the rose has $2n$ petals. If n is odd, the rose has n petals.

$r = a\sin 2\theta$
Rose curve with 4 petals

$r = a\cos 3\theta$
Rose curve with 3 petals

$r = a\cos 4\theta$
Rose curve with 8 petals

$r = a\sin 5\theta$
Rose curve with 5 petals



Lemniscates

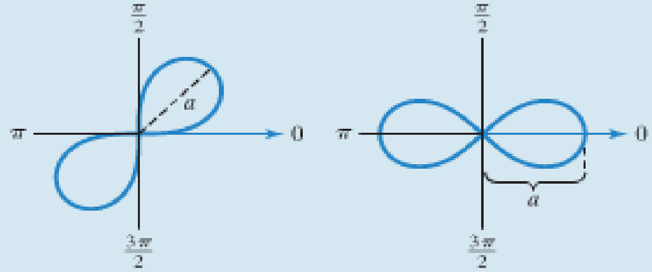
The graphs of

$$r^2 = a^2\sin 2\theta \quad \text{and} \quad r^2 = a^2\cos 2\theta, \quad a \neq 0$$

are called **lemniscates**.

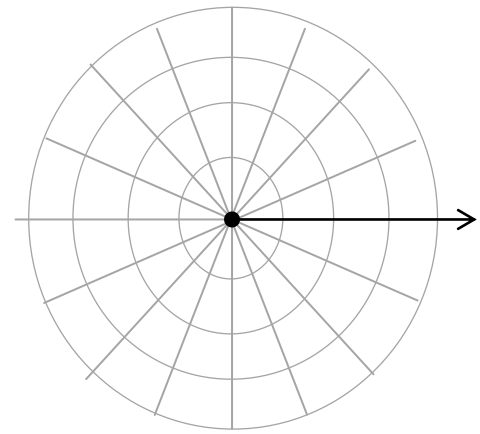
$r^2 = a^2\sin 2\theta$ is symmetric with respect to the pole.

$r^2 = a^2\cos 2\theta$ is symmetric with respect to the polar axis, $\theta = \frac{\pi}{2}$, and the pole.

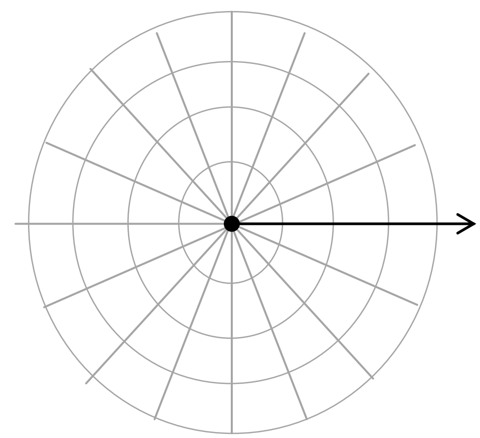


Examples: Graph the following…

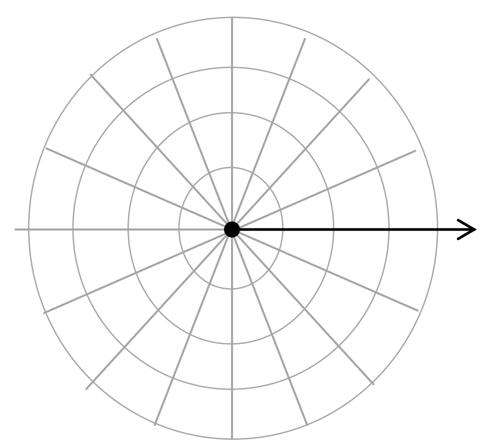
$r = 1 - 2 \sin \theta$



$r = 3 \cos 2\theta$



$r^2 = 4 \cos 2\theta$



Example: Test for symmetry about the polar axis on $r = 2 + \cos \theta$

Symmetric Definitely not symmetric Maybe symmetric