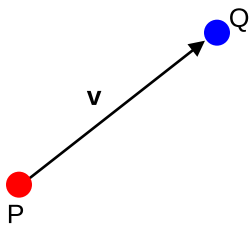
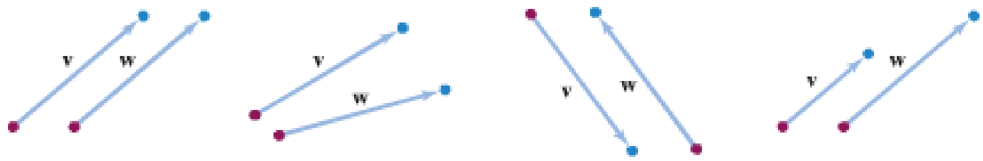
<u>**Section 6.6**</u> <u>(Vectors)</u>

Quantities that involve both a direction and magnitude are called **vectors**. Consider someone driving due north at 45 mph. The *<u>magnitude</u>* would be the speed (45 mph) and the *<u>direction</u>* would be the direction (due north or 90^o)

Some quantities can be described using only magnitude (such as the temperature of the room), and these are called *<u>scalars</u>*.

We will concentrate on directed line segments where the magnitude of this vector is its length and the direction is the angle or slope. For 2 vectors to be equal, they must have the same magnitude and direction. Consider the vector **v** = $\overrightarrow{PQ}$ and vector W below





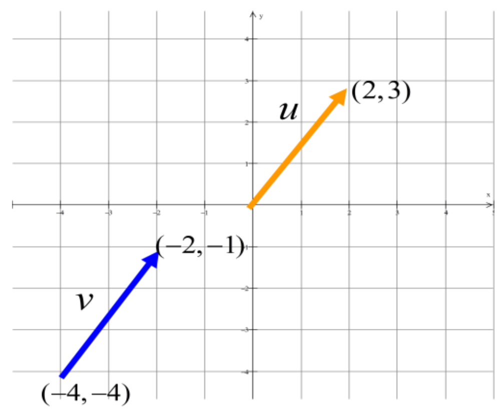
(a) v = w because the vectors have the same magnitude and same direction.

(b) Vectors v and w have the same magnitude, but different directions.

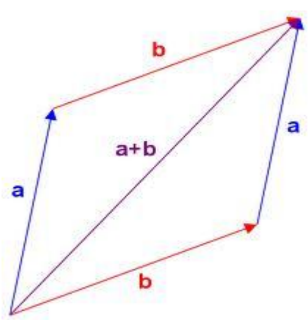
(c) Vectors v and w have the same magnitude, but opposite directions.

(d) Vectors v and w have the same direction, but different magnitudes.

<u>Example</u>: Show that **u** = **v** in the picture on the right



A vector can be multiplied by a real number (scalar). The effect of scalar multiplication is a change in vector magnitude (direction shows no change except that the direction reverses if multiplying by a negative number)



When adding 2 vectors **a** and **b**, you essentially place the initial point of vector **b** on the terminal point of vector **a** and examine the resultant vector formed by connecting the initial point of **a** with the terminal point of **b** (or vice versa)

Vector subtraction is similarly found (consider **a** – **b** = **a** + (– **b**) and see figure 6.54)

In the rectangular coordinate system, we use vectors with magnitude 1 in the x and y-directions (vector **i** lies along the x-axis and vector **j** lies along the y-axis). This allows us to represent a vector **v** in the rectangular coordinate system using **i** and **j** (vector v from (0,0) to point (a,b) is **v** = a**i** + b**j** with magnitude (||**v**|| = $\sqrt{a^2 + b^2}$)

<u>Example</u>: Sketch vector **v** = 3**i** – 3**j** on the board and find its magnitude

With vectors not starting at the origin, vector **v** can be expressed as **v** = $(x_2 - x_1)$**i** + $(y_2 - y_1)$**j**

<u>Example</u>: Let **v** be the vector from initial point $P_1 = (-1, 3)$ to terminal point $P_2 = (2, 7)$. Write **v** in terms of **i** and **j**.

Book problems:

With vectors expressed in terms of **i** and **j**, we can easily perform vector addition, subtraction, scalar multiplication.

Example: If **v** = 7**i** + 3**j** and w = 4**i** – 5**j**, find each of the following vectors…

v + **w** **v** – **w** 8**v** –5**w** 6**v** – 3**w**

The zero vector has magnitude 0 and is assigned no direction (**0** = 0**i** + 0**j**).

A unit vector has magnitude 1, and it is often helpful to find the unit vector that has the same direction as a given vector (this is used frequently in applications). For any nonzero vector **v** , the vector **v** / ||**v**|| (**v** divided by its magnitude) is the unit vector that has the same direction as **v**.

Example: Find the unit vector in the same direction as **v** = 4**i** – 3**j** and verify that it has magnitude 1

Let **v** = a**i** + b**j** be a nonzero vector with direction angle θ from the positive x-axis to **v.** The vector can be expressed in terms of its magnitude and direction angle as **v** = ||**v**|| cos θ **i** + ||**v**|| sin θ **j**

A common vector that represents the direction and speed of an object in motion is called a velocity vector

Example: The jet stream is blowing at 60 miles per hour in the direction of N 45^{o} E. Express its velocity as a vector

Another common vector that represents forces acting on an object is a force vector. See example of holding a box in figure 6.61 and page 707 in the book.

The resultant force of 2 (or more) forces acting on an object is the vector sum of all forces. We can find the magnitude and direction of the resultant force after finding the resultant force vector…

Example: Two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ of magnitude 30 and 60 pounds act on an object. The direction of $\mathbf{F_1}$ is N10^{o}E and the direction of $\mathbf{F_2}$ is N60^{o}E. Find the magnitude and direction angle of the resultant force (see example 9 – pg. 707)

Example (bonus if no time): Awesome quarterback Logan Thomas releases a football with a speed of 50 feet per second at an angle of 30^{o} with the horizontal. Express this using a vector.

See #'s 75, 76 in book (pg. 710)

Book problems: