Section 6.7 (The Dot Product)

In the last section, we examined vector addition and scalar multiplication, which resulted in vectors. In this section, we examine the **dot product** of two vectors, which results in a scalar value. If **v** = a**i** + b**j** and **w** = c**i** + d**j** are vectors, the dot product is defined as

v • w = ac + db

…the sum of the products of their horizontal components and their vertical components.

Example: If **v** = 7**i** – 4**j** and **w** = 2**i** – **j** , find **v • w** , **w • v** , and **w • w**

As seen in the example, dot products of vectors is commutative (**v • w** = **w • v**). Other properties are listed on the right.

The Law of Cosines can be used to derive another formula for the dot product that gives us a way to find the angle between 2 vectors.

Considering the triangle formed by vectors **v** and **w** below and applying the Law of Cosines gives ||**u**|| = ||**v**||^2 + ||**w**||^2 – 2||**v**||||**w**||cosθ

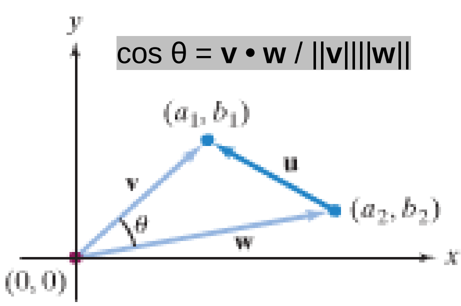
Using the horizontal and vertical components for each vector and substitution we can simplify to **v • w** = ||**v**||||**w**||cos θ (see pg. 714)

Properties of the Dot Product

If **u**, **v**, and **w** are vectors, and c is a scalar, then

1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
2. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
3. $\mathbf{0} \cdot \mathbf{v} = 0$
4. $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$
5. $(c\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{u} \cdot \mathbf{v}) = \mathbf{u} \cdot (c\mathbf{v})$

cos θ = **v • w** / ||**v**||||**w**||



Example: Find the angle between the vectors **v** = 4**i** – 3**j** and **w** = **i** + 2**j**

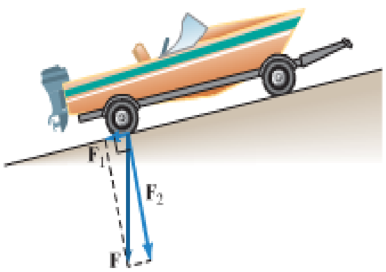
Two vectors are parallel when the angle between the vectors is 0^o or 180^o (point in the same or opposite directions). Two vectors are called orthogonal when the angle between them is 90^o (vectors meet at right angles).

Example: What is the dot product of 2 vectors **v** and **w** that are orthogonal?

The converse of this dot product is also true. If the dot product of **v • w** is _______, the vectors are orthogonal.

Example: Are the vectors **v** = 2**i** + 3**j** and **w** = 6**i** – 4**j** orthogonal?

Book problems:



We've examined how to add up 2 vectors to get a resultant vector, but what if we wanted to do the reverse? Consider a boat on a tilted ramp. The force due to gravity (**F**) is pulling straight down on the boat. Part of this force (**F_1**) is pushing the boat down the ramp while another part of this force is pushing the boat down against the ramp at a right angle to the incline (**F_2**). These 2 orthogonal vectors are called the vector components of **F** (**F** = **F_1** + **F_2**). A method for finding **F_1** and **F_2** involves projecting a vector onto another vector.

The vector projection of vector **v** onto vector **w** is $\text{proj}_{\mathbf{w}}\,\mathbf{v} = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\,\mathbf{w}$ (see explanation and graphs on pgs. 716-717 and example 4 for more details)

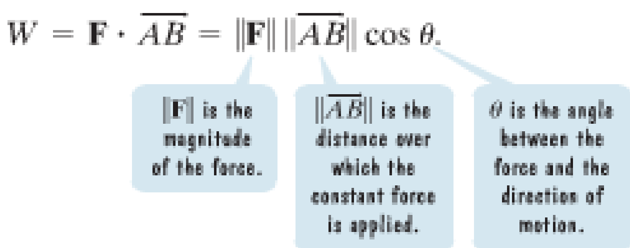
Example: If **v** = 2**i** – 5**j** and **w** = **i** – **j** find the projection of **v** onto **w**

Let **v** and **w** be 2 nonzero vectors. Vector **v** can be expressed as the sum of 2 orthogonal vectors **v_1** and **v_2** where **v_1** is parallel to **w** and **v_2** is orthogonal to **w**

$$\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\,\mathbf{v} = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\,\mathbf{w} \qquad\qquad \mathbf{v}_2 = \mathbf{v} - \mathbf{v}_1$$

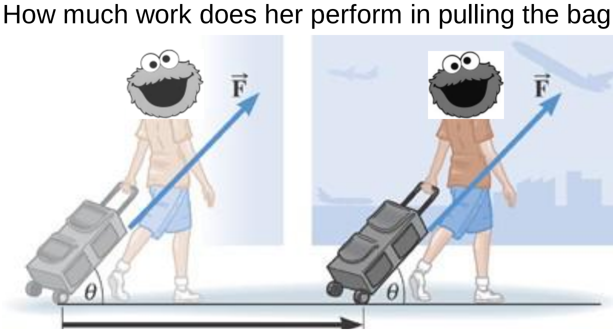
Example: Let **v** and **w** be the vectors defined in the previous example. Decompose **v** into 2 vectors **v_1** and **v_2** , where **v_1** is parallel to **w** and **v_2** is orthogonal to **w**

Work – The work (W) done by a force (F) in moving an object from A to B is (see pgs. 718 and 719)…



Consider the work you would do in pulling a suitcase (or book-bag) down a hallway (notice that there are 2 forces being applied – x and y or horizontal and vertical – this ties in with vector decomposition)…

Example: Cookie monster has loaded his bag full of cookies, and is trying to drag it down the hall to Trigonometry class (ground is level). He applies a force of 30 pounds on a handle that makes an angle of 50° with the ground. How much work does her perform in pulling the bag 150 feet?



Book problems: