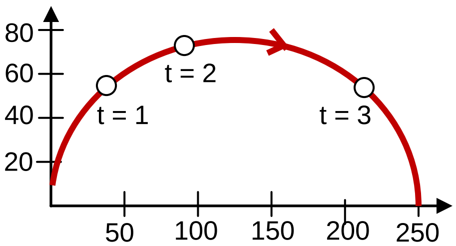
Section 9.5 (Parametric Equations)

In a recent game against the Barney's, Ernie threw a baseball from an initial height of 4 feet, with an initial velocity of 90 feet per second and at an angle of 40^o with the horizontal. After t seconds the location of the ball can be described by…

$x = (90 \cos 40^o)t$ - ball's horizontal distance in feet - - AND -
$y = 4 + (90 \sin 40^o)t - 16t^2$ - ball's vertical distance in feet –

Using these equations as time (t) changes, we can calculate the location of the ball at any time.

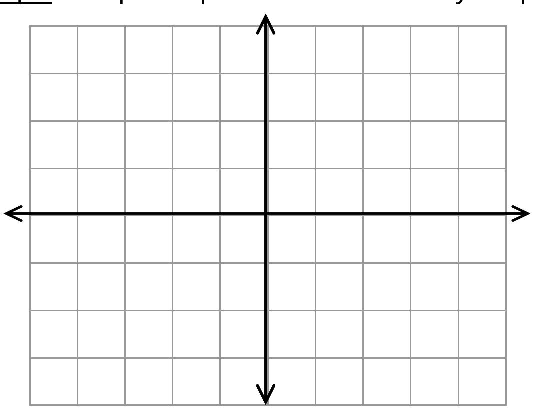
This set of equations (x and y as functions of t) are called **parametric equations**



Time (t)	x(t) (horizontal)	y(t) (vertical)
0	0 ft.	4 ft.
1	$90 \cos 40^o$ ~ 69 ft.	~ 46 ft.
2	~ 138 ft.	~64 ft.
3	~ 207 ft.	~34 ft.
3.6	~248 ft.	~0 ft.

A **plane curve** is the set of ordered pairs (x,y) where x = f(t) and y = g(t) for t in an interval I and the equations x = f(t) and y = g(t) are called **parametric equations** for the curve

Example: Graph the plane curve defined by the parametric equations $\mathbf{x = t^2 + 1}$, $\mathbf{y = 3t}$, $\mathbf{-2 \le t \le 2}$



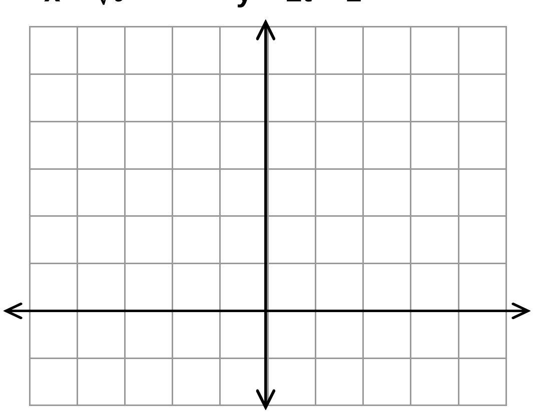
We examined a variety of points in the first example to graph a plane curve. Another method is to eliminate the parameter (t) to write one equation in x and y equivalent to the two equations (see discussion on pg. 927).

1. Solve for t in one of the parametric equations
2. Substitute the expression for t in the other equation

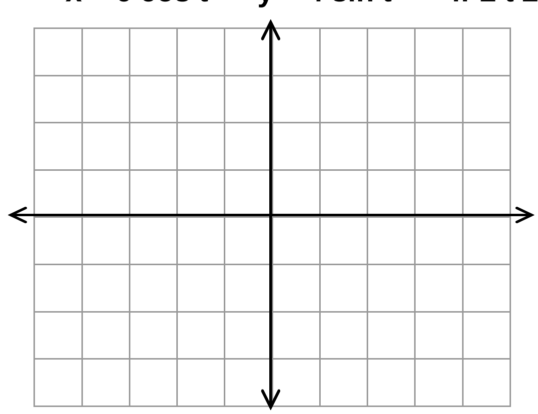
(you may need to change the domain of the rectangular equation to be consistent with the domain for the parametric equation)

Example: Sketch the plane curve represented by the following parametric equations by eliminating the parameter
(use identity $\sin^2 t + \cos^2 t = 1$)

$\mathbf{x = \sqrt{t}}$ $\mathbf{y = 2t - 1}$



$\mathbf{x = 6 \cos t}$ $\mathbf{y = 4 \sin t}$ $\mathbf{\pi \le t \le 2\pi}$



It is sometimes helpful in applications to find a set of parametric equations for a given function (there are often multiple possible sets of parametric equations for a given function, but remember that the parametric equations must allow x to take on all the values in the domain of the original equation. One method if y = f(x) is to let x = t and go from there…

Example: Find a set of parametric equations for the parabola whose equation is $\mathbf{y = x^2 - 25}$

See cycloid application in the book for details on a different application problem, but let's explore our first example a bit more.

The path of a projectile launched h feet above the ground with an initial velocity of v_0 ft. per second and at an angle θ with the horizontal is given by

$x = (v_0 \cos \theta)t$ - object's horizontal distance in feet - - AND -
$y = h + (v_0 \sin \theta)t - 16t^2$ - object's vertical distance in feet –

Example: A trigonometry book is launched by an angry student with an initial velocity of 200 ft. per second at an angle of 42^o with the horizontal. The book was launched from a height of 4 feet.

a) Find the parametric equations that describe the position of the book as a function of time.

b) Describe the book's position after t = 1 second and after t = 3 seconds

c) How long (to the nearest tenth of a sec.) is the book in flight, and how far does the book fly (horizontally)?