Section 1.7 (Combinations of Functions and Composite Functions)

- Before exploring combinations of functions, let's review the domain of a function
  - The domain of a function f(x) is the set of real numbers for which the value of f(x) is defined
  - In particular, we need to exclude numbers that cause ***division by zero*** in a rational function and numbers that result in the ***root (even root) of a negative number***
    - Because x/0 and sqrt(-1) are undefined (not real numbers)

Examples: Find the domain of each function (what values of x make this function work)

$f(x) = x^2 + 3x - 17$ $\quad\quad$ $g(x) = \dfrac{5x}{x^2 - 49}$ $\quad\quad$ $h(x) = \sqrt{9x - 27}$

- Given 2 functions, the sum, difference, product, and quotient of those functions are also functions
- The domain for these combined functions consist of all real numbers that are common to the domains of the 2 individual functions (except for sometimes the quotient)

Example: Research has shown that the equation A(x) = 2x – 15 represents the number of games Alabama typically wins in a season and B(x) = x – 2 represents the number of games Auburn wins (x represents the number of games played). How many games combined will Alabama and Auburn win if they play 10 games?

A(10) = $\quad\quad$ B(10) = $\quad\quad$ A(10) + B(10) = $\quad\quad$ (A + B)(10) =

- Given functions f and g, we have (the domain of these combinations consists of numbers that are in the domain of each of the functions)

Sum: (f + g)(x) = f(x) + g(x) $\quad\quad$ Difference: (f – g)(x) = f(x) – g(x)

Product: (fg)(x) = f(x) * g(x) $\quad\quad$ Quotient: $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$

Examples: If f(x) = x + 3 and g(x) = 3x – 1, find the following (and the domain in each case)

(f + g)(x) = $\quad\quad$ (f – g)(x) =

(f * g)(x) = $\quad\quad$ (f/g)(x) =

- To find the composition of functions (f ◦ g)(x) = f(g(x)), you basically plug the function g(x) into the function f(x) and vice versa

(composite functions are used when the output of one function is used as the input to another function as in applying discounts while shopping – see explanations on pp. 224-225 in the book)

- The domain of a composite function f(g(x)) is the set of all numbers for x such that x is in the domain of g and g(x) is in the domain of f

Book problems: 3,5,9,17,19,35,39,43,45,57,61,63,65,71,73,75,77,91

Example: At Hooper's store on Sesame Street, Virginia Tech jerseys are finally going on sale to make room for the shipment of the newly designed jerseys. Jerseys are already $10 off, and you can take an additional 20% off of that reduced price. If you find a men's jersey originally marked at $70 and a women's jersey originally marked at $60, what would be the sale price of each jersey?

Example: If $f(x) = 2x + 3$, find f(☺)

Examples: If $f(x) = x^2$ and $g(x) = 2x + 1$, find each composition

$f(g(3)) =$

$g(f(3)) =$

$f(g(x)) = (f \circ g)(x) =$

$g(f(x)) =$

Examples: If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $(f \circ g)(x)$ and $(g \circ f)(x)$

Example: Express the given function h as a composition of 2 functions f and g so that $h(x) = f(g(x))$ where one of the functions is $3x + 4$.

$h(x) = (3x + 4)^3$

$h(x) = \sqrt{3x + 4}$

Look at online HW problems (especially the last couple) in class

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