Section 1.8 (Inverse Functions)

Suppose Oscar the Grouch has decided to offer a big sale on his trashy computers. He says that all laptops will be $300 off. Putting this into function form, if the laptop's original price is represented by x, then we would have f(x) = _______________. Unfortunately for Oscar, the tabloids discovered that just prior to the sale, Oscar had jacked up the prices on all of his computers by $300 anyway. His price increase is represented by g(x) = x + 300.

Example: Find f(g(x)) and g(f(x)) based on these functions.

Given 2 functions f(x) and g(x), **if f(g(x)) = x** for every x in the domain of g **and g(f(x)) = x** for every x in the domain of f, then the function g is the **inverse** of f and can be denoted as g(x) = $\mathbf{f^{-1}(x)}$

Examples: Find f(g(x)) and g(f(x)) and determine if the given functions are inverses of each other.

$f(x) = 9x$ and $g(x) = \dfrac{x}{9}$ $\qquad\qquad$ $f(x) = 4x - 7$ and $g(x) = \dfrac{x+7}{4}$

For a one-to-one function not defined as a set of ordered pairs as above, you can find the inverse by

1. Replace f(x) with y
2. Interchange x and y
3. Solve for y
4. Replace y with $f^{-1}(x)$

Example: Find the equation of the inverse of

$f(x) = \dfrac{3}{x} - 1$ $\qquad\qquad$ $g(x) = 2x + 3$ $\qquad\qquad$ $h(x) = x^3 - 3$

Book problems: 1,3,7,15,17,21,23,27,29,31,41,43,47

Recall that for a relationship to be a function, each element in the domain (x-value) had to correspond to only one element in the range (y-value). For a given function to be **one-to-one** (and have an inverse), the reverse must also be true (each y-value must correspond to only one x-value).

Example: Determine whether each function described is one-to-one.

$f = \{ (7,3) , (-1,1) , (5,0) , (4,-2) \}$

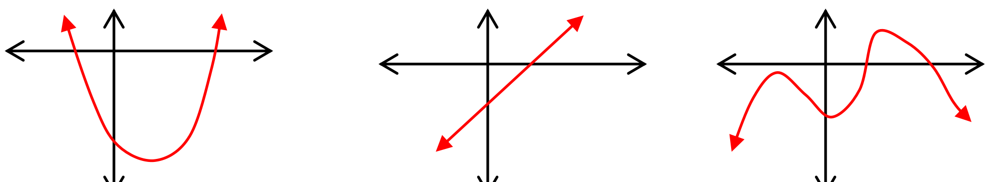
Domain Range

$g = \{ (-3,2) , (6,3) , (2,14) , (-6,2) \}$

Domain Range

Similarly to the vertical line test for functions, a horizontal line test can determine if a given function is one-to-one (why?)

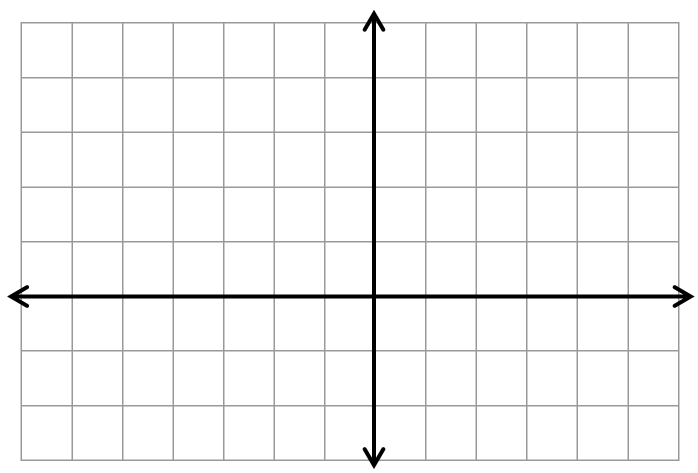
Example: Which of the following have inverse functions?



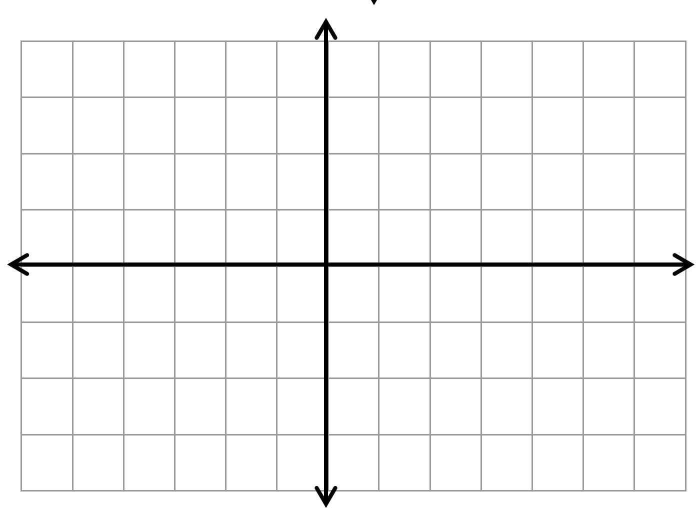
The graph of f^{-1} is the graph of f reflected about the line y = x (since we are simply switching x and y) -show example on the board

Example: Find the inverse of the following (if it exists), graph the function and its inverse and give the domain and range of the function

$f(x) = (x+2)^3$



$g(x) = \sqrt{x+1} \quad x \geq 0$



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