**Section 2.1** (Complex Numbers)

In this section we travel to an imaginary place called the ***complex number system***

* The imaginary unit i is used to denote the square root of -1 ( i2 = -1 and= i)

Using i, we can express the square root of any number ( $\sqrt{-25}=\sqrt{-1(25)}=\left(\sqrt{-1}\right)\sqrt{25}=5i$)

The complex number system contains all real numbers and can be written as **a + bi**

Examples: The following are examples of complex numbers

 **– 4 + 6i** **2i = 0 + 2i** **3 = 3 + 0i**

a, the real part is 3

b, the imaginary part is 2

a, the real part is 0

b, the imaginary part is 0

b, the imaginary part is 6

a, the real part is -4

Complex numbers can be manipulated as follows (given a + bi and c + di as complex numbers)

* **a + bi = c + di** if and only if the real parts are equal and the imaginary parts are equal (a=b & c=d)
* sum => add the real parts and then add the imaginary parts => **(a+bi) + (c+di) = (a+c) + (b+d)i**
* difference => same as sum but with subtraction => **(a+bi) – (c+di) = (a-c) + (b-d)i**

Examples: Add / Subtract

 **(5 + 2i) + (4 – 3i) = 6i – (2 – i) =**

 **(–2 – 4i) + (–3) = (3 + 4i) – (2 + 3i) =**

Multiplying complex numbers is similar to polynomials, replacing occurrences of **i2** with **-1**

Examples: Multiply (remember the FOIL method?)

 **– 5i \* 3i = – 2i (6 – 2i) = (3 – 4i)(6 + i) =**

 **(1 – 2i)2 = (6 + 5i)(6 – 5i) =**

Complex numbers **(a + bi)** and **(a – bi)** are called ***complex conjugates*** of each other and

**(a + bi)(a – bi) = a2 + b2** (real) **(a - bi)(a + bi) = a2 + b2** (real)

We use complex conjugates to divide a complex number (multiply top and bottom by the conjugate of the denominator to eliminate imaginary number in the denominator)

Examples: Divide (write answer in the standard form a + bi)

 $\frac{3+i}{2-3i}$ $\frac{7+4i}{2-5i}$

Keep in mind that the product rule for radicals doesn’t necessarily hold true for ***imaginary numbers*** (when multiplying, always write numbers in terms of i first)

NO  YES

**When performing operations with square roots of negative numbers, begin by expressing all square roots in terms of i**

Examples: Perform the indicated operations and write the result in standard form

 $\left(-2+\sqrt{-3}\right)^{2}$ $\frac{-14+\sqrt{-12}}{2}$

Recall that a quadratic equation can be expressed in the general form as ax2 + bx + c = 0, a 0. Also, all quadratic equations can be solved by the ***quadratic formula***



This sometimes results in imaginary numbers that are complex conjugates.

Example: Solve using the quadratic formula

 **x2 – 2x + 2 = 0 7x2 = 2x – 1**