**Section 2.3** (Polynomial Functions and Their Graphs)

(Remember that you can always pick a point for x and solve to help graph)

The function defined by



is a ***polynomial*** of ***degree n*** (the largest of the exponents) if n (the ***exponents***) are ***nonnegative integers*** and an , an-1 , … a2 , a1 , a0 (the coefficients) are real numbers.

The number an , the coefficient of the variable to the highest power, is called the ***leading coefficient***

Examples: Determine if the following are polynomials. If so, state the degree. If not, tell why.

f(x) = -6x + x9 g(x) = 6x - h(x) = x2 + 2x + x1/2 - 4

f(x) = 3x + 4 g(x) = 7 h(x) = 

Polynomial functions of degree 2 or higher have graphs that are smooth (rounded curves – no sharp corners) and continuous (no breaks and can be drawn without lifting pencil / pen) – note: what about degree 1 and degree 0?

Examples: Are the following graphs of polynomial functions?

Consider the trend of f(x) = 2x4 - 80x

|  |  |  |  |
| --- | --- | --- | --- |
| Trend | x | f(x) = 2x4 – 80x | Result |
| x is small  (all terms have an impact) | 0 | f(x) = 0 – 0 = 0 | Graph falls from 0 to negative |
| 2 | 2(16) – 80(2) = -128 |
| x increases slightly  (all terms still have some impact) | 5 | 2(625) – 80(5) = 625 | Graph rises right |
| 6 | 2(1296) – 80(6) = 2112 |
| x is large  (the leading term dominates like the Hokies in football) | 100 | 2(1000000000) – 80(100) = 199992000 | Graph rises steeply |
| … | Really big number |

As x increases / decreases without bound, the graph of the polynomial function



eventually rises or falls depending on the leading term (term with the largest degree – the coefficient and the variable part).

The key is to look at the degree of the variable and the sign of the coefficient in the leading term

* Degree n is odd
  + Leading coefficient is positive => Graph falls to the left and rises to the right
  + Leading coefficient is negative => Graph rises to the left and falls to the right
* Degree n is even
  + Leading coefficient is positive => Graph rises on both sides
  + Leading coefficient is negative => Graph falls on both sides

Examples: Look at the chart on page 304 and use the Leading Coefficient Test to determine the end behavior of the graph of the following

f(x) = 3x7 – 2x5 – 4x2 + 6 g(x) = 8x6 + 4x5 + 8 h(x) = -4x4 + 3

The values of x for which the polynomial function f(x) is equal to 0 are called the ***zeros*** of f (also called the ***roots*** or ***solutions*** of f). Each real root is an x-intercept of the graph of the polynomial function.

Example: Find all zeros of…

f(x) = x3 + 2x2 – 4x – 8 g(x) = x4 – 4x2

In the 2nd example just shown, the factorization was g(x) = x2(x + 2)(x – 2). The factor x occurs twice and the factors (x + 2) and (x – 2) occur once. The number of times a factor occurs gives the ***multiplicity*** of that zero. So our solution 0 has multiplicity 2 and our solutions 2 and -2 have multiplicity 1.

If a zero (solution) has ***even multiplicity***, the graph touches the x-axis at this point and turns around. If a zero (solution) has ***odd multiplicity***, the graph crosses the x-axis at this point.

Example: Find the zeros / the multiplicity of each zero, and state the behavior of the graph at each zero

g(x) = x4 – 4x2 f(x) = -4(x + ½)2(x – 5)3

You can use the facts learned in this section and other steps mentioned (pg. 310) to sketch graphs of polynomial functions (note additional steps of finding y-intercept by finding f(0) and determining possible symmetry by looking at f(-x) = f(x) : y-axis symmetry, f(-x) = -f(x) : origin symmetry)

Examples: After reviewing the chart and steps on pg. 310, use those steps to graph x3 – 3x2 on the board