**Section 2.5** (Zeros of Polynomial Functions)

The Rational Zero Theorem (pg. 327) allows us to make a list of all possible zeros (solutions) of a polynomial function with integer coefficients

Possible rational zeros = Factors of the constant term (on the end)\_\_\_\_\_\_

Factors of the leading coefficient (the beginning)

Example: List all possible rational zeros for f(x) = x3 + 2x2 – 5x – 6

Constant term = Factors =

Leading coefficient = Factors =

Possible zeros = { }

Given those rational zeros, we can then use synthetic division to determine a legitimate zero for the function.

Example: Test the above possible zeros using synthetic division to find an actual zero

1 | 1 2 -5 -6

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Since \_\_\_\_\_ is a zero / solution (because of the remainder of 0), then x - \_\_\_\_\_ = **x + 1** is a factor or

f(x) = x3 + 2x2 – 5x – 6 = (x + 1)(**x2 + x – 6**) = (x + 1)(x + 3)(x – 2)

--- using results from synthetic division ---

Therefore the solution set of f(x) = x3 + 2x2 – 5x – 6 or the zeros of the given f(x) are { }

Example: Solve…

x3 + 8x2 + 11x – 20 = 0 x3 + x2 – 5x – 2 = 0

Example: Find the zeros of f(x) = x4 – 6x3 + 22x2 – 30x + 13

Properties of roots of polynomial equations:

1. A polynomial of degree n has n roots (counting multiple roots separately)
2. If a + bi is a root to a polynomial equation (b 0), then the conjugate a – bi is also a root

Notice in the above example, we had solutions of {1, 2 3i} which looks to be only 3 roots; however, our factorization was (x – 1)2(2 3i) = (x – 1)(x – 1)(2 + 3i)(2 – 3i), so by counting those factors separately, we indeed have 4 roots for the 4th degree equation.

Using the above properties and the Linear Factorization Theorem (pg. 333), we can now find polynomials if given certain information.

Example: Find a 3rd degree polynomial function f(x) with real coefficients such that 2 and 5i are zeros and f(-1) = -234 (write answer in standard form)

By Descartes’ Rule of Signs (pg. 334), the number of positive and negative real roots possible within a polynomial function f(x) can be examined by looking at the sign changes within f(x) and f(-x)

Example: Determine the possible numbers of positive and negative real zeros of f(x) = -7x3 + 4x2 – 7x + 4

f(x) = -7x3 + 4x2 – 7x + 4 f(-x) =

\_\_\_\_ sign changes

\_\_\_\_ or \_\_\_\_\_\_\_

Possible positive zeros