**Section 2.6** (Rational Functions and Their Graphs)

Rational functions are quotients of polynomial functions that can be expressed as

 , where p and q are polynomial functions and q(x) 0

Example: Find the domain of each rational function (all real #’s where the denominator does not equal 0)

   

 {x | x \_\_\_\_\_\_\_}

Given the function, we see that x = 2 is not in the domain of f(x), but what happens as x approaches x = 2?

x approaches 2 from left (denoted as x 2 - ), f(x) \_\_\_\_\_\_

x = 2

 f(0) = -1/2 f(1) = -1 f(1.99) = -100 …

x approaches 2 from right (denoted as x 2 + ), f(x) \_\_\_\_\_\_

 f(4) = f(3) = f(2.001) = …

What happens to the above graph and f(x) as x ? as x – ?

Look at HW example with time

Notice how the graph above approaches but never touches the line x = 2. A line x = a (such as this) is called a ***vertical asymptote*** when f(x) increases or decreases without bound as x approaches a.

Given ***simplified*** rational function, if a is a zero of q(x), then x = a is a vert. asymptote of f(x)

Example: Find the vertical asymptotes, if any, of the given rational function…

   

Using the graph above again, we see that as x gets bigger and bigger, f(x) approaches \_\_\_\_\_. A line y = b (such as this) is called a ***horizontal asymptote*** if f(x) b as x increases or decreases without bound.

Given a rational function where the degree of the numerator is n and the degree of the denominator is m…

* If n < m, the x-axis (or y = 0) is the horizontal asymptote of the graph
* If n = m, the line y = leading coefficient of the numerator / leading coefficient of the denominator is the horizontal asymptote of the graph
* If n > m, the graph has no horizontal asymptote

Example: Find the horizontal asymptote, if any, of the graphs of the following…

   

 Numerator deg. =

 Denominator deg. =

The book lays out a seven step method for graphing rational functions (p. 347)

1. Determine symmetry => f(-x) = f(x) : y-axis symmetry, f(-x) = -f(x) : origin symmetry
2. Find y-intercept
3. Find x-intercept(s)
4. Find vertical asymptote(s)
5. Find horizontal asymptote(s)
6. Plot points between x-intercepts and vertical asymptotes (and other points if desired)
7. Graph the function

Examples: Use this method to graph the following…

  

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|   |  |  |   |   |   |   |   |   |   |   |   |   |   |
|   |  |  |   |   |   |   |   |   |   |   |   |   |   |
|   |  |  |   |   |   |   |   |   |   |   |   |   |   |
|   |  |  |   |   |   |   |   |   |   |   |   |   |   |
|   |  |  |   |   |   |   |   |   |   |   |   |   |   |
|   |  |  |   |   |   |   |   |   |   |   |   |   |   |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|   |  |  |   |   |   |   |   |   |   |   |   |   |   |
|   |  |  |   |   |   |   |   |   |   |   |   |   |   |