**Section 2.8** (Modeling Using Variation)

When you swim underwater, the pressure in your ears depends on the depth at which you swim. The formula *p = 0.43d* describes the water pressure, *p*, in pounds per square inch (psi), at a depth of *d* feet.



At a depth of 10 feet (d = 10), we have p = 0.43(10) = 4.3 psi

At a depth of 20 feet (d = 20), we have p = 0.43(10) = 8.6 psi

At a depth of 80 feet (d = 80), we have p = 0.43(80) = 34.4 psi

The formula *p = 0.43d* illustrates that the water pressure is a constant multiple of your underwater depth. Here, the pressure *p* varies directly as the depth *d* (if *d* doubles, *p* doubles, etc.) – see book…

In functions, y **varies directly** as x (y is **directly proportional** to x) if there’s a nonzero constant k such that ***y = kx*** (constant k is called the constant of variation or the constant of proportionality) -- (note that these variations provide a linear relationship)

Examples: Suppose y varies directly with x. If y is 24 when x is 8, find y when x is 6.

Example: The number of gallons of water, W, used when taking a shower varies directly with time t. A shower lasting 5 min. used 30 gallons of water. How much water is used in a shower lasting 11 minutes?

Example: Hooke’s Law – The distance a spring stretches is directly proportional to the weight attached to the spring. If a spring stretches 8 inches when a 56 lb. weight is attached to it, find the distance that an 35 lb. weight stretches the spring.



d

When y is proportional to the inverse of a variable x, we have y =and say that y **varies inversely** with x or y is **inversely proportional** to x

Examples: Suppose y varies inversely with x. If y is 6 when x is 3, find y when x is 9.

Example: The speed r at which one needs to drive in order to travel a constant distance is inversely proportional to the time t. If I can travel to Blacksburg to watch the Hokies whip Miami in 8 hours at 50 mph, how fast will I need to drive in order to make the trip in 5 hours?

Note that we can also say that a variable y varies directly with the square (or cube, etc.) of x as y = kx2

Example: The distance, s, that a body falls from rest varies directly as the square of the time, t, of the fall. If skydivers fall 64 feet in 2 seconds, how far will they fall in 4 seconds?

In combined variation, direct and inverse variation can occur at the same time.

Example: If y varies directly as x and inversely as z, and y = 16 when x = 32 and z = 2, find y when x = 27 and z = 3.

In joint variation, a variable y can vary directly as the product of 2 or more variables.

Example: The area of a triangle varies jointly as its base and height. Express the area in terms of base b and height h. Then solve for b.

Example: Now y varies jointly as x and z. y = 25 when x = 2 and z = 5. Find y when x = 8 and z = 12.

Example: Body-mass index (BMI) varies directly as one’s weight (in pounds) and inversely as the square of one’s height (in inches). A person who weighs 180 pounds and is 5 feet (60 inches) tall has a BMI of 35.15. What is the BMI for a 170-pound person who is 5 feet 10 inches tall?



Example: Write an equation that expresses the following relationship: x varies directly as m and inversely as the difference between y and a. Then solve the equation for y.