Section 3.2 (Logarithmic Functions)

In the last section we defined an **exponential function** $f(x) = b^x$ where b is a positive constant other than 1 and x is any real number. Let's try to define the inverse of this function.

$y = b^x$ (switch x and y) => $x = b^y$ (solve for y) => ???

The inverse function of the exponential function with base b is the **logarithmic function** with base b

$f(x) = \log_b x$ is the logarithmic function with base b where $y = \log_b x$ is equivalent to $b^y = x$

There are 2 ways to express the same equation here

Logarithmic Form: $y = \log_b x$ (Exponent: y; Base: b)

Exponential Form: $b^y = x$ (Exponent: y; Base: b)

Example: Write the following equations in equivalent exponential form

$3 = \log_7 x$ $\log_3 9 = y$ $3 = \log_b 64$

Example: Write the following equations in equivalent logarithmic form

$2^5 = x$ $b^3 = 27$ $\frac{1}{16} = 2^{-4}$ $\sqrt{9} = 3$

Given a logarithmic expression, you really ask yourself one question to evaluate the expression…

$\log_3 9$ means 3 to what power gives me 9? => because $3^{\mathbf{2}} = 9$ then $\log_3 9 = \mathbf{2}$

$\log_{25} 5$ means 25 to what power gives me 5? => because $25^{??} = 5$ then $\log_{25} 5 =$ ____

Examples: Evaluate:

$\log_4 64$ $\log_5 \frac{1}{125}$ $\log_{36} 6$ $\log_3 \sqrt[5]{3}$ $\log_9 9$ $\log_8 1$

Re-examining the inverse of the exponential function ($f(x) = b^x$), we can now go a little further…

$y = b^x$ (switch x and y) => $x = b^y$ (solve for y) => $\mathbf{\log_b x = y}$

This gives us the inverse properties of logarithms $\log_b b^x = x$ and $b^{\log_b x} = x$

Examples: Evaluate $\log_7 7^8$ and $3^{\log_3 17}$

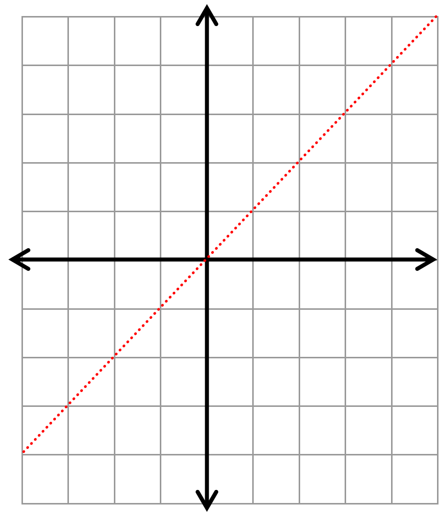
Considering that logarithmic functions are the inverses of exponential functions we know that their graphs should be symmetric about the line y = x…

Example: Graph $f(x) = 3^x$ and $g(x) = \log_3 x$ in the given coordinate system…

Book problems: 7,11,13,29,37,39,43,47,53,55,63,75,83,97,119

x	$f(x) = 3^x$
-2	
-1	
0	
1	
2	

x	$g(x) = \log_3 x$



Vertical Asymptote =>

<u>REVIEW CHARACTERISTICS OF GRAPHS</u> (pgs. 404-405)

Table 3.4 on pg. 405 discusses transformations of these logarithmic graphs

<u>Examples</u>: Use these properties to describe the graphs of $f(x) \log_5 (x + 3)$ and $g(x) = 5 – \log_2 x$

<u>REVIEW ONLINE HW PROBLEMS 8-10</u>

In looking at the domain of logarithmic functions, consider that in $y = \log_b x$ that we said b is a positive constant. So, if a positive number is raised to any power, the sign of the result is _______________.

The domain of a logarithmic function of the form $f(x) = \log_b x$ is the set of all positive real numbers

<u>Example</u>: Find the domain of $f(x) = \log_4 (x – 5)$

The logarithmic function with base 10 is called the *common logarithmic function* and the function $f(x) = \log_{10} x$ is usually expressed simply as $f(x) = \log x$ (the LOG button on calculators refers to the common log or $\log_{10} x$).

<u>Example</u>: Consider some properties of the common log (pg. 407) and use these properties to find $\log 10^6$

Furthermore, the logarithmic function with base e is called the natural logarithmic function and is usually expressed as $f(x) = \ln x$ (see pgs. 408-410 for properties and note the inverse properties).

<u>Examples</u>: Review online HW problems 13-14.

Book problems: 7,11,13,29,37,39,43,47,53,55,63,75,83,97,119