**Section 4.2** (The Unit Circle)

The unit circle is the circle with radius 1 and center at the origin of the rectangular coordinate system. The equation of the unit circle is given by \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Arc length = s = rθ** = 1(θ) = θ (radian measure of the central angle is the same as the length of the intercepted arc)

**θ**

One trigonometric function is defined as **y =** r **sin θ** (pronounced sine of θ)

This is a function of θ and you can see that the input to the function is θ (the angle in radians) and the output involves y (the y-coordinate of the point on the unit circle that corresponds to angle θ

**(1,0)**

Trigonometric functions defined…

**y =** r **sin θ** (note r=1 in unit circle, giving **y = sin θ**) **csc θ = 1 / y, y 0**

**x =** r **cos θ sec θ = 1 / x, x 0**

**y / x =** r sin θ / r cos θ = **tan θ , x 0 cot θ = x / y, y 0**

Example: Find the values of the 6 trigonometric functions for the angle θ corresponding to the point 

 

 sin θ = y = csc θ =

**(1,0)**

**θ**

 cos θ = sec θ =

 tan θ = cot θ =

Example: Find the same values at P = (0 , –1)

We can use these formulas to find the values of trigonometric functions at t = π / 4, remembering that the formula for our unit circle is x2 + y2 = 1 and knowing that the point at this angle has x = y (sketch and work on board)

The point P = (a , b) on the unit circle corresponding to t = π / 4 is (\_\_\_\_\_\_\_\_\_\_\_ , \_\_\_\_\_\_\_\_\_\_\_\_\_)

Example: Use this point to find sin π / 4, csc π / 4, and cos π / 4

Consider whether sin θ and cos θ are even or odd functions by examining sin(–θ) and cos(–θ)

sin θ = y , sin(– θ) =

**– θ**

**(1,0)**

cos θ = x , cos(– θ) =

**θ**

Cosine and secant functions are \_\_\_\_\_\_\_\_\_\_\_\_\_ (cos(–θ) = cos θ, sec(–θ) =sec θ)

Sine, cosecant, tangent, cotangent are \_\_\_\_\_\_\_\_\_ (sin(–θ) = – sin θ, tan(–θ) = – tan θ)

Example: Find the value of each of the following trigonometric functions

  

Since sin θ = y and csc θ = 1 / y, we also have csc θ = 1 / sin θ (we can use similar techniques to find)…

Reciprocal Identities Quotient Identities

  

Example: Given **sin θ =**  and **cos θ =**, find the value of the remaining 4 trig functions…

We can explore other relationships among the trig identities by examining the equation of the unit circle

 x2 + y2 = 1 => (cos θ)2 + (sin θ)2 = 1 => sin2 θ + cos2 θ = 1 (common form)

 Pythagorean Identities: **sin 2 θ + cos 2 θ = 1 1 + tan 2 θ = sec 2 θ 1 + cot 2 θ = csc 2 θ**

Example: Given that **sin θ =** $\frac{1}{2}$ and **0 ≤ θ ≤** $\frac{π}{2}$, find the value of cos t using a trig identity

Certain patterns in nature repeat again and again over a given period of time (tides every 12 hours for example). Trigonometric functions are often used to model periodic things. Consider a point (x , y) on the unit circle that travels along the perimeter over an angle of 2π radians (draw on board).

For any integer n and real number (angle) θ => sin(θ + 2πn) = sin θ and cos(θ + 2πn) = cos θ

sin, cos, csc, and sec have period 2π. What about tan and cot (consider the same example and look at x and y coordinates after travelling over an angle of π radians).

For any integer n and real number (angle) θ => tan(θ + πn) = tan θ tan has period π

Example: Find the value of  and 

Confirm that you can use a calculator to evaluate trigonometric functions (sin π / 4 and csc 1.5)