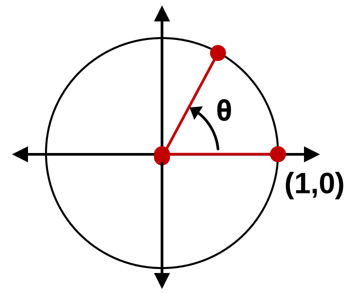
Section 4.2 (The Unit Circle)

The unit circle is the circle with radius 1 and center at the origin of the rectangular coordinate system. The equation of the unit circle is given by ________________________________

Arc length = s = rθ = 1(θ) = θ (radian measure of the central angle is the same as the length of the intercepted arc)



One trigonometric function is defined as **y =** r **sin θ** (pronounced sine of θ)

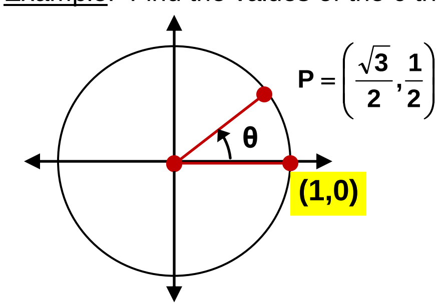
This is a function of θ and you can see that the input to the function is θ (the angle in radians) and the output involves y (the y-coordinate of the point on the unit circle that corresponds to angle θ

Trigonometric functions defined…

y = r **sin θ** (note r=1 in unit circle, giving **y = sin θ**)
x = r **cos θ**
y / x = r sin θ / r cos θ = **tan θ , x $\neq$ 0**

csc θ = 1 / y, y $\neq$ 0
sec θ = 1 / x, x $\neq$ 0
cot θ = x / y, y $\neq$ 0

Example: Find the values of the 6 trigonometric functions for the angle θ corresponding to the point $P = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



sin θ = y =

cos θ =

tan θ =

csc θ =

sec θ =

cot θ =

Example: Find the same values at P = (0 , –1)

We can use these formulas to find the values of trigonometric functions at t = π / 4, remembering that the formula for our unit circle is $x^2 + y^2 = 1$ and knowing that the point at this angle has x = y (sketch and work on board)

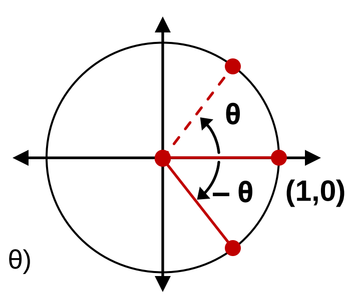
The point P = (a , b) on the unit circle corresponding to t = π / 4 is (____________ , ______________)

Example: Use this point to find sin π / 4, csc π / 4, and cos π / 4

Consider whether sin θ and cos θ are even or odd functions by examining sin(–θ) and cos(–θ)

sin θ = y , sin(– θ) =

cos θ = x , cos(– θ) =



Cosine and secant functions are ______________ (cos(–θ) = cos θ, sec(–θ) =sec θ)

Sine, cosecant, tangent, cotangent are __________ (sin(–θ) = – sin θ, tan(–θ) = – tan θ)

Example: Find the value of each of the following trigonometric functions

$\sec\left(-\frac{\pi}{4}\right) =$ $\sin\left(-\frac{\pi}{4}\right) =$

Since sin θ = y and csc θ = 1 / y, we also have csc θ = 1 / sin θ (we can use similar techniques to find)…

Reciprocal Identities

$$\sin(\theta) = \frac{1}{\csc(\theta)} \quad \csc(\theta) = \frac{1}{\sin(\theta)}$$
$$\cos(\theta) = \frac{1}{\sec(\theta)} \quad \sec(\theta) = \frac{1}{\cos(\theta)}$$
$$\tan(\theta) = \frac{1}{\cot(\theta)} \quad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Quotient Identities

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$
$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Example: Given $\sin\theta = \frac{2}{3}$ and $\cos\theta = \frac{\sqrt{5}}{3}$, find the value of the remaining 4 trig functions…

We can explore other relationships among the trig identities by examining the equation of the unit circle

$x^2 + y^2 = 1$ => $(\cos\theta)^2 + (\sin\theta)^2 = 1$ => $\sin^2\theta + \cos^2\theta = 1$ (common form)

Pythagorean Identities: $\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

Example: Given that $\sin\theta = \frac{1}{2}$ and $0 \le \theta \le \frac{\pi}{2}$, find the value of cos t using a trig identity

Certain patterns in nature repeat again and again over a given period of time (tides every 12 hours for example). Trigonometric functions are often used to model periodic things. Consider a point (x , y) on the unit circle that travels along the perimeter over an angle of 2π radians (draw on board).

For any integer n and real number (angle) θ => $\sin(\theta + 2\pi n) = \sin\theta$ and $\cos(\theta + 2\pi n) = \cos\theta$

sin, cos, csc, and sec have period 2π. What about tan and cot (consider the same example and look at x and y coordinates after travelling over an angle of π radians).

For any integer n and real number (angle) θ => $\tan(\theta + \pi n) = \tan\theta$ tan has period π

Example: Find the value of $\cot\left(\frac{5\pi}{4}\right)$ and $\cos\left(-\frac{9\pi}{4}\right)$

Confirm that you can use a calculator to evaluate trigonometric functions (sin π / 4 and csc 1.5)