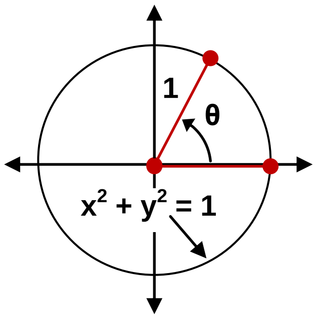
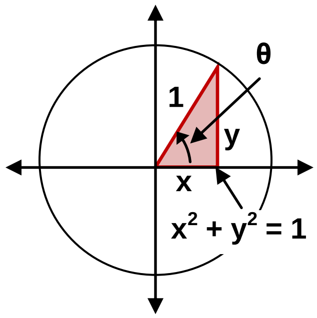
Section 4.3 (Right Triangle Trigonometry)

Trigonometry (measurement of triangles) is used in navigation, building, and engineering, among other applications as we will see throughout this section. Consider the angle **$\pi/3$** on the unit circle as examined in the last section. Now construct a right triangle by dropping a line segment from our point perpendicular to the x-axis.



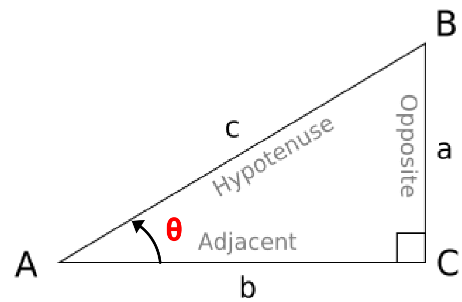


Right Triangle Definitions of Trig. Functions

Using this right triangle with angle **t** and the knowledge that y = sin(t) and x = cos(t), we see that sin(**θ**) = **y** = **y/1** and cos(**θ**) = **x** = **x/1**

In many applications it is helpful to interpret trigonometric functions using these right triangles. Using the triangle below, construct the six trigonometric functions …

The six trigonometric functions of acute angle θ are defined as

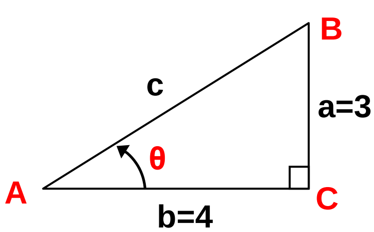


sin θ	= $\frac{\text{opposite}}{\text{hypotenouse}}$	= $\frac{a}{c}$	csc θ	= $\frac{\text{hypotenouse}}{\text{opposite}}$	= $\frac{c}{a}$
cos θ	= $\frac{\text{adjacent}}{\text{hypotenouse}}$	= $\frac{b}{c}$	sec θ	= $\frac{\text{hypotenouse}}{\text{adjacent}}$	= $\frac{c}{b}$
tan θ	= $\frac{\text{opposite}}{\text{adjacent}}$	= $\frac{a}{b}$	cot θ	= $\frac{\text{adjacent}}{\text{opposite}}$	= $\frac{b}{a}$

One way to remember is via SOHCAHTOA (**S**ome **O**ld **H**okies **C**ame **A**round **H**ere and **T**ook **O**ut **A**la-bleh-ma)

The trigonometric values of angle θ are the same despite how large the triangle is (see similar triangles on pg. 491)

Example: Find the value of each of the 6 trigonometric functions of θ in the figure



sin θ =

csc θ =

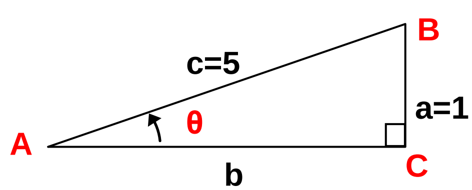
cos θ =

sec θ =

tan θ =

cot θ =

Example: Find the value of each of the 6 trigonometric functions of θ in the figure (simplify answers)



In section 4.2, we used points on the unit circle to evaluate the trigonometric identities of a 45° or $\pi/4$ angle. We can similarly determine the trigonometric identities by constructing a right triangle with a 45° angle (see figure 4.34).

Example: If a triangle has angles of 90° and 45°, what must the other angle measure (how many total degrees are in a triangle)?

Example: Given that this is an isosceles triangle (2 equal angles and 2 equal sides), if we let the 2 equal legs have a length of 1, what will be the length of the other side?

Example: Sketch triangle on board and determine trigonometric values of 45° or $\pi/4$ angle

Book problems:

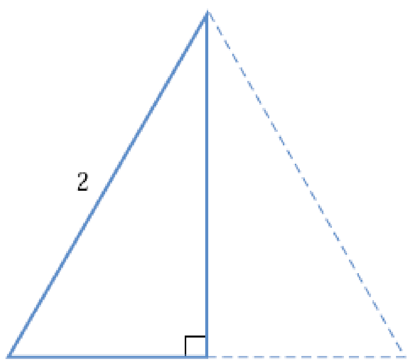


Figure 4.26 A 30°-60°-90° triangle

Two other angles commonly found throughout trigonometry are 30^o and 60^o (π/6 and π/3 angle). We can derive information on these angles by starting with an equilateral triangle (3 equal angles and sides) and dividing it into 2 right triangles.

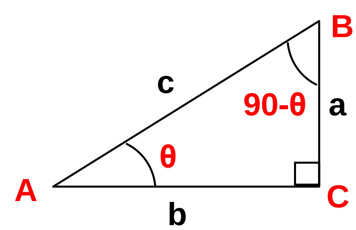
Example: Completely label the triangle on the left

Example: Find sin 30^o, cos 60^o, tan 30^o, and tan 60^o

You will need to be able to construct 45-45-90 and 30-60-90 triangles from memory by recalling these (practice on board with time)

Two angles are considered complements if their sum is 90^o or π/2. Refer to the right triangle below which has 2 acute angles that are complements (θ and 90-θ).

Example: Find sin θ and cos (90-θ)



Because sin θ = cos (90-θ) and vice versa, sin and cos are called **cofunctions**

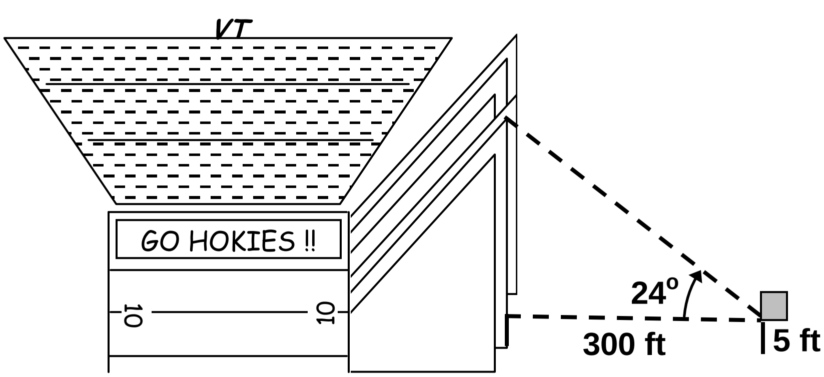
Cofunction Identities

sin θ = cos (90-θ) --- cos θ = sin (90-θ)
tan θ = cot (90-θ) --- cot θ = tan (90-θ)
sec θ = cos (90-θ) --- csc θ = sin (90-θ)

Example: Find a cofunction with the same value as the given expression: sin (**46^o**) cot (**π/12**)

Many applications of right triangle trigonometry involve the angle made with an imaginary horizontal line. An angle formed by a horizontal line and the line of sight to an object above the horizontal line is called the **angle of elevation** (below the horizontal is called the **angle of depression**). – SEE FIGURE 4.37 – pg. 495

Example: Sighting the top of Lane Stadium, a surveyor measured the angle of elevation to be 24^o. The transit (measuring platform) is 5 feet above the ground and 300 horizontally from the stadium. Find the stadium's height.



If two sides of a right triangle are known, you can use inverse trigonometric functions to find an acute angle θ

Example: Use you calculator to find θ where sin θ = 0.866

Example: The flagpole in the middle of Sesame Street is 14 meters tall and casts a shadow 10 meters long. Sketch the problem and find the angle of elevation of the sun to the nearest degree.

Book problems: