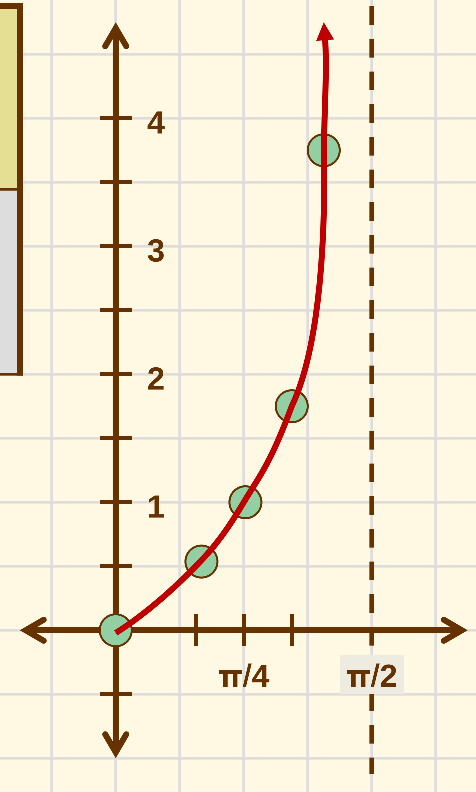
Graphs of Other Trigonometric Functions

- The tangent function has some properties that are different than the sinusoidal trig. functions, resulting in a graph that differs significantly
  - The tangent function is undefined at π/2 (and – π/2, etc.)
  - The period of the tangent function is π
  - The tangent function is an odd function (meaning f(-x) = -tan x = -f(x) = -tan x and that the graph is symmetric with respect to the origin)
- Consider the following table of values on the graph of y = tan x

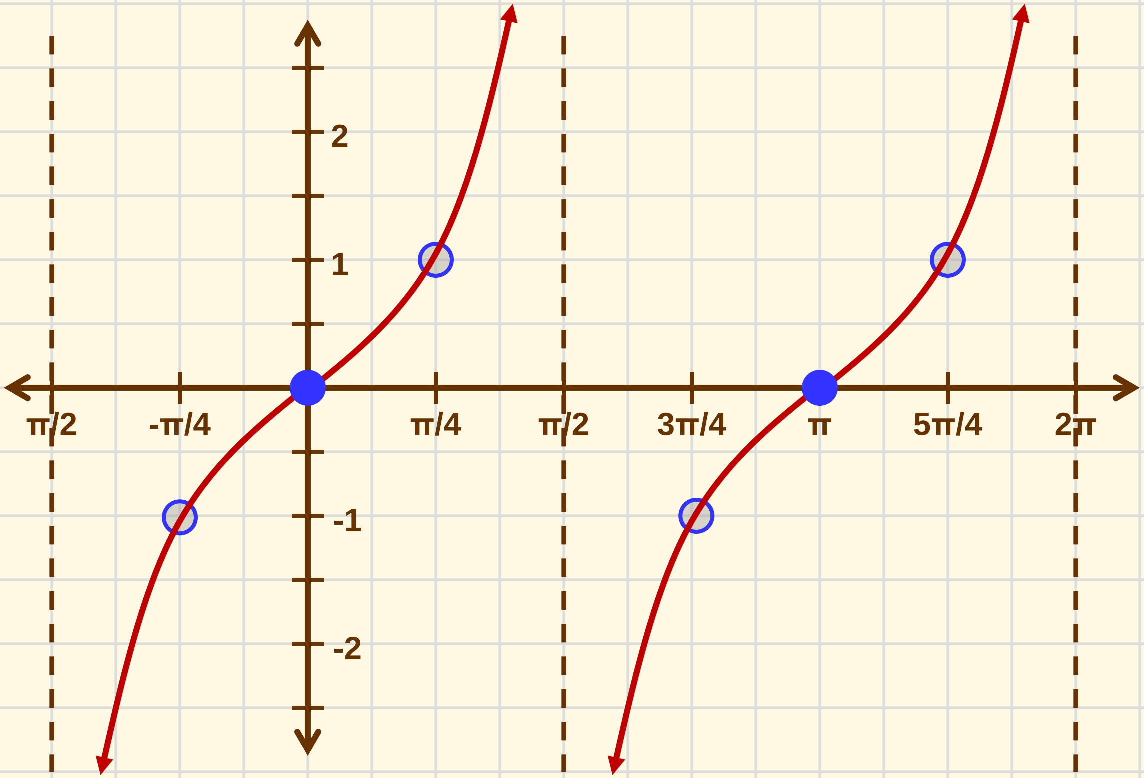
x	**0**	**π/6 (0.52)**	**π/4 (0.79)**	**π/3 (1.05)**	**5π/12 (1.31) (75°)**	**89π /180 (1.55) (89°)**	**π/2 (1.57) (90°)**
y = tan x							

The graph of y = tan x has a vertical asymptote at x = π/2



Graphs of Other Trigonometric Functions

- The graph of y = tan x can be completed on the interval (– π/2, π/2) using the origin symmetry or by examining more points



Tangent curve characteristics

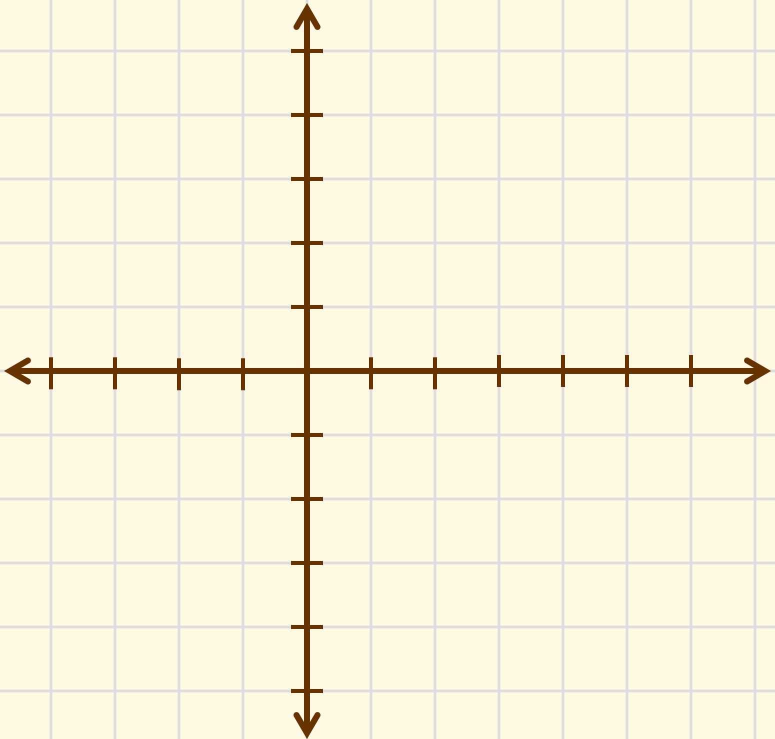
- Period = π
- Vertical Asymptotes occur at odd multiples of π / 2
- Odd function with origin symmetry
- An x-intercept occurs midway between asymptotes
- y-values of -1 and 1 occur halfway between x-intercept and asymptotes

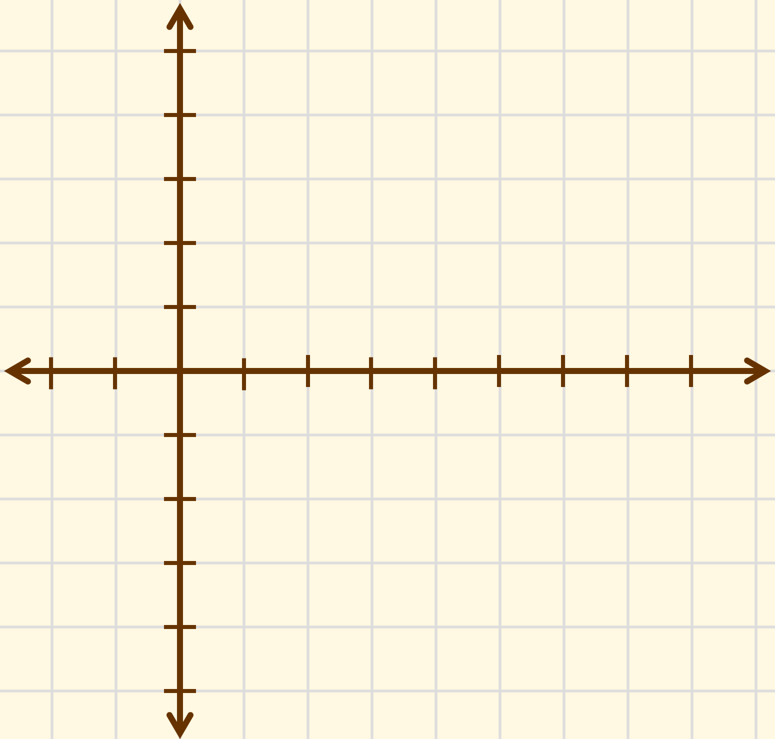
Graphs of Other Trigonometric Functions

- We can use similar techniques as in the last section to look at variations of the tangent function graph y = A tan (Bx – C)

1. Find 2 consecutive vertical asymptotes ($-\frac{\pi}{2} < Bx - C < \frac{\pi}{2}$ implies Bx – C = ____ and Bx – C = ____
2. Identify x-intercept (halfway between asymptotes)
3. Find points on graph ¼ and ¾ of way between asymptotes (y-coordinates here should be –A and A, respectively)
4. Steps 1-3 graph one full period of the function (add additional cycles to right / left as needed)

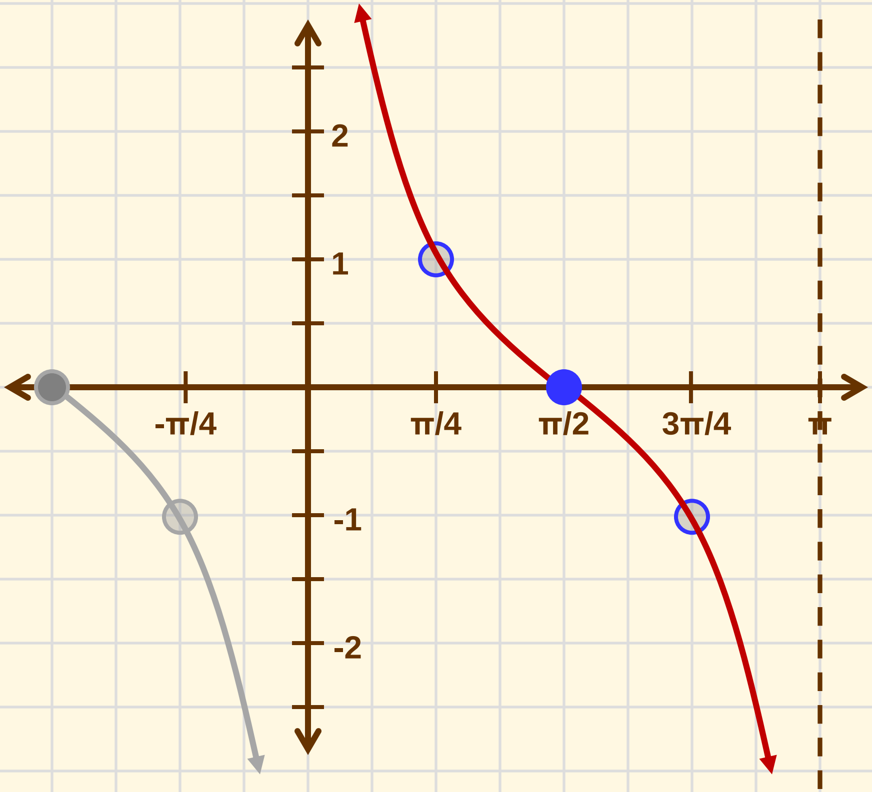
- Examples: Graph y = 3 tan 2x for $-\frac{\pi}{4} < x < \frac{3\pi}{4}$ -and- 2 full periods of $y = \tan\left(x - \frac{\pi}{2}\right)$





Graphs of Other Trigonometric Functions

- The graph of y = cot x is similar to that of tan x but is completed on the interval (0, π) and is flipped vertically (moves down when going from left to right)



Cotangent curve characteristics

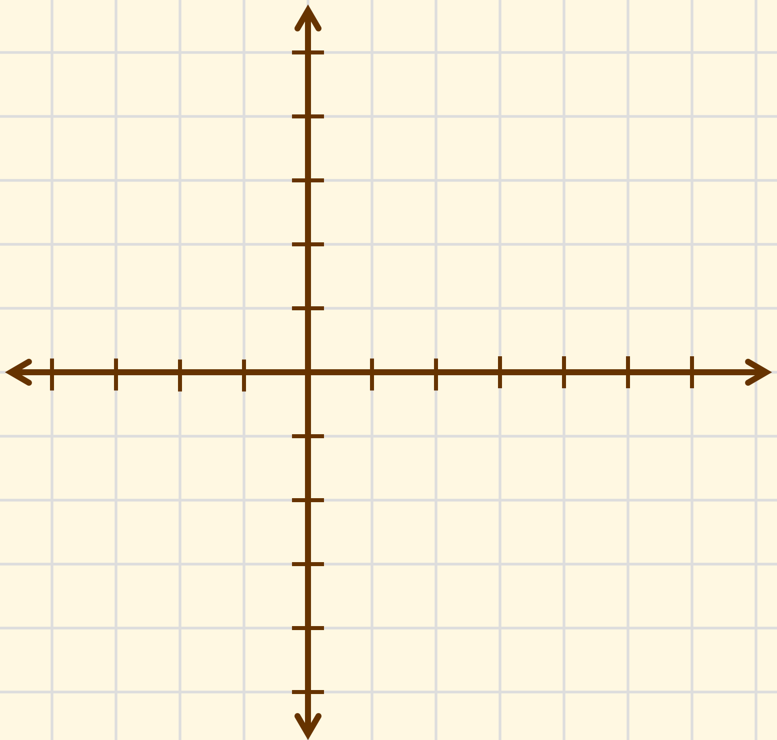
- Period = π
- Vertical Asymptotes occur at **integral** multiples of **π**
- Odd function with origin symmetry
- An x-intercept occurs midway between asymptotes
- y-values of -1 and 1 occur halfway between x-intercept and asymptotes

Graphs of Other Trigonometric Functions

- We can use similar techniques as in tangent to look at variations of the cotangent function graph $y = A \cot (Bx - C)$

  1. Find 2 consecutive vertical asymptotes ($0 < Bx - C < \pi$ implies $Bx - C =$ ____ and $Bx - C =$ ____
  2. Identify x-intercept (halfway between asymptotes)
  3. Find points on graph ¼ and ¾ of way between asymptotes (y-coordinates here should be $-A$ and A, respectively)
  4. Steps 1-3 graph one full period of the function (add additional cycles to right / left as needed)

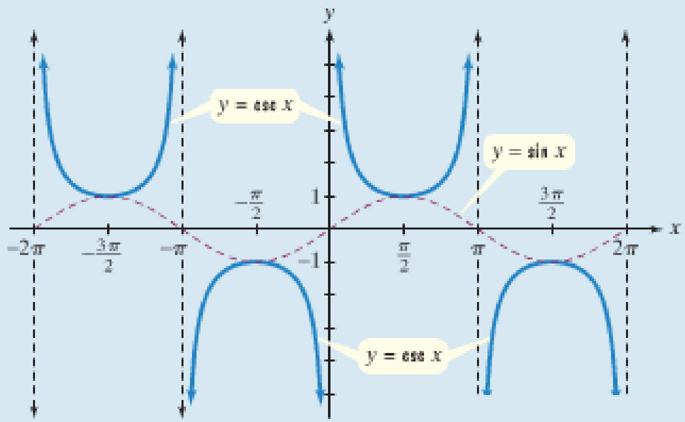
- Examples: Graph $y = \frac{1}{2} \cot \frac{\pi}{2}x$



Graphs of Other Trigonometric Functions

- We can use the fact that csc x and sec x are the reciprocals of sin x and cos x to examine those graphs (using x-values of sin an cos, the corresponding y-values are simply the reciprocals of the y-values for sin and cos)
- Consider the characteristics (notice the dashed red line indicating sin and cos)…

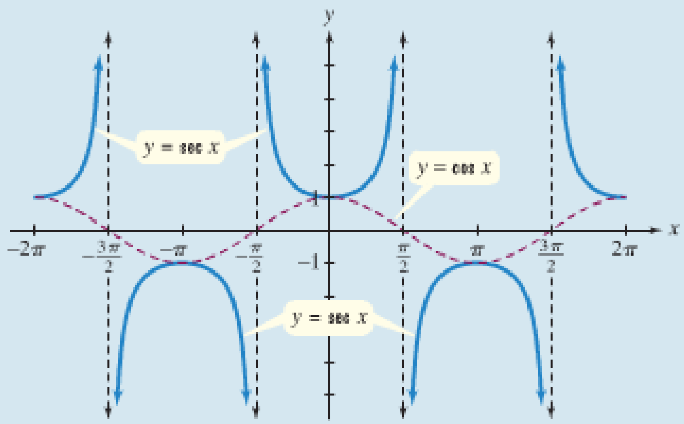
The Cosecant Curve: The Graph of $y = \csc x$ and Its Characteristics



Characteristics

- **Period:** 2π
- **Domain:** All real numbers except integral multiples of π
- **Range:** All real numbers y such that $y \le -1$ or $y \ge 1$: $(-\infty, -1] \cup [1, \infty)$
- **Vertical asymptotes** at integral multiples of π
- **Odd function,** $\csc(-x) = -\csc x$, with origin symmetry

The Secant Curve: The Graph of $y = \sec x$ and Its Characteristics



Characteristics

- **Period:** 2π
- **Domain:** All real numbers except odd multiples of $\frac{\pi}{2}$
- **Range:** All real numbers y such that $y \le -1$ or $y \ge 1$: $(-\infty, -1] \cup [1, \infty)$
- **Vertical asymptotes** at odd multiples of $\frac{\pi}{2}$
- **Even function,** $\sec(-x) = \sec x$, with y-axis symmetry

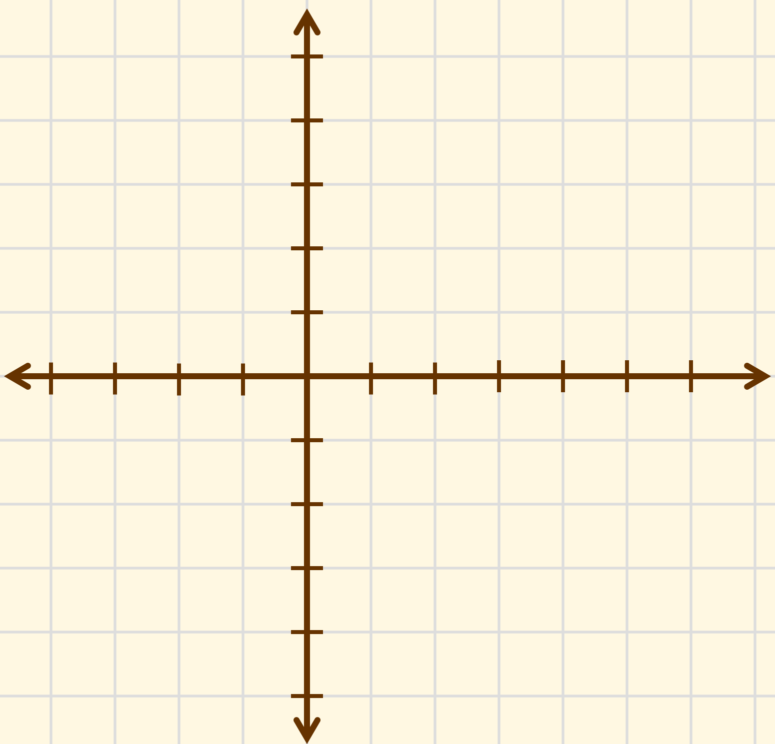
Graphs of Other Trigonometric Functions

- We can use the graphs of sin x and cos x to help graph csc x and sec x

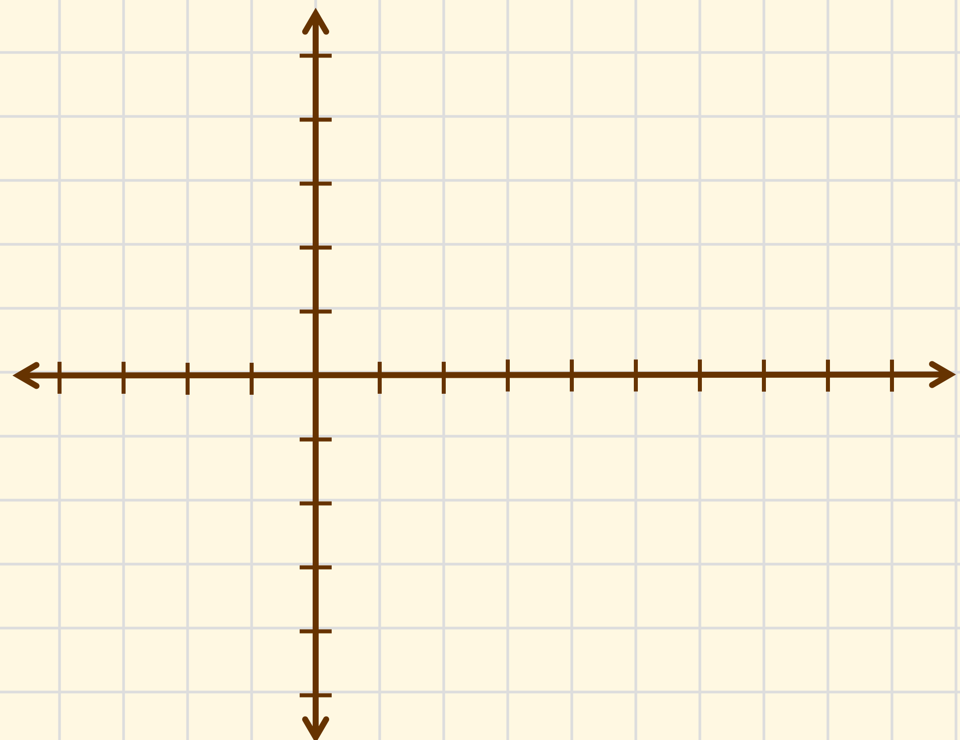
csc / sec curve characteristics

- x-intercepts of sin (cos) curve correspond to vertical asymptotes of csc (sec) curve
- maximum point on sin (cos) curve corresponds to minimum on csc (sec) curve
- minimum point on sin (cos) curve corresponds to maximum on csc (sec) curve

Example: Graph y = sin 2x and then graph its reciprocal function y = csc 2x



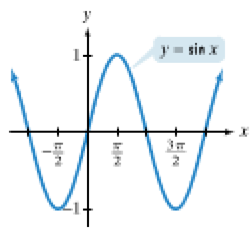
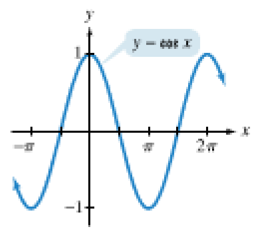
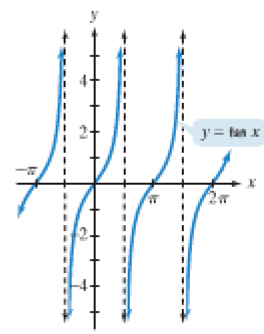
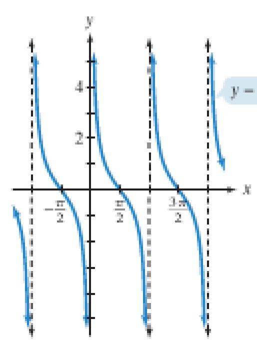
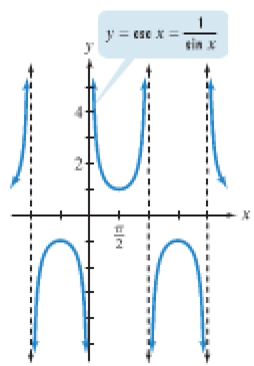
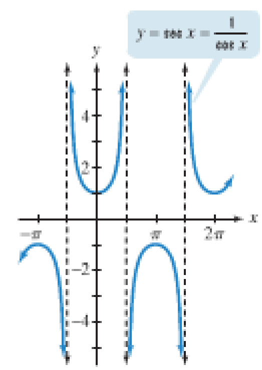
Example: Graph y = 2 sec 2x for $-\frac{3\pi}{4} < x < \frac{3\pi}{4}$



Graphs of Other Trigonometric Functions

- Below are characteristics of the six basic trigonometry functions (review)…

Table 4.6 Graphs of the Six Trigonometric Functions

 **Domain:** all real numbers: $(-\infty, \infty)$ **Range:** $[-1, 1]$ **Period:** 2π	 **Domain:** all real numbers: $(-\infty, \infty)$ **Range:** $[-1, 1]$ **Period:** 2π	 **Domain:** all real numbers except odd multiples of $\frac{\pi}{2}$ **Range:** all real numbers **Period:** π
 **Domain:** all real numbers except integral multiples of π **Range:** all real numbers **Period:** π	 **Domain:** all real numbers except integral multiples of π **Range:** $(-\infty, -1] \cup [1, \infty)$ **Period:** 2π	 **Domain:** all real numbers except odd multiples of $\frac{\pi}{2}$ **Range:** $(-\infty, -1] \cup [1, \infty)$ **Period:** 2π