

Section 5.1 (Verifying Trigonometric Identities)

How many of you enjoy puzzles? Verifying trigonometric identities is similar to solving puzzles.

Using fundamental trigonometric identities learned in the previous chapter can help in these verification problems.

<u>Reciprocal</u>	<u>Quotient</u>	<u>Pythagorean</u>	<u>Even-Odd</u>
$\sin(t) = \frac{1}{\csc(t)}$ $\csc(t) = \frac{1}{\sin(t)}$ $\cos(t) = \frac{1}{\sec(t)}$ $\sec(t) = \frac{1}{\cos(t)}$ $\tan(t) = \frac{1}{\cot(t)}$ $\cot(t) = \frac{1}{\tan(t)}$	$\tan(t) = \frac{\sin(t)}{\cos(t)}$ $\cot(t) = \frac{\cos(t)}{\sin(t)}$	$\sin^2 t + \cos^2 t = 1$ $1 + \tan^2 t = \sec^2 t$ $1 + \cot^2 t = \csc^2 t$	$\sin(-x) = -\sin(x)$ $\cos(-x) = \cos(x)$ $\tan(-x) = -\tan(x)$ $\csc(-x) = -\csc(x)$ $\sec(-x) = \sec(x)$ $\cot(-x) = -\cot(x)$

To verify an identity, show that one side of the identity is identical to the other. No one technique can be used to verify all identities. It often helps to rewrite one side (or both) so that it contains only sin and cos, though.

Example: Verify the identity $\csc x \tan x = \sec x$

When verifying identities by manipulating one side, continue to look at the other side to remember the desired result

Example: Verify the identity $\cos x \cot x + \sin x = \csc x$

Sometimes you can use factoring to help simplify a trigonometric expression.

Example: Verify the following identities

$$\sin x - \sin x \cos^2 x = \sin^3 x$$

$$\frac{1 + \cos \theta}{\sin \theta} = \csc \theta + \cot \theta$$

Also remember techniques of working with rational expressions to combine / eliminate fractions with the LCD

Example: Verify the following identities

$$\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = 2 \csc x$$

$$\frac{\cos x}{1+\sin x} = \frac{1-\sin x}{\cos x}$$

Sometimes it can be helpful to work each side of the identity separately and then show they are equal

Examples: Verify the following identities

$$\frac{\sec x + \csc(-x)}{\sec x \csc x} = \sin x - \cos x$$

$$\frac{1}{1+\sin \theta} + \frac{1}{1-\sin \theta} = 2 + 2 \tan^2 \theta$$

See page 593 for general guidelines and tips on verifying trigonometric identities and do ONLINE HW

Guidelines for Verifying Trigonometric Identities

- Work with each side of the equation independently of the other side. Start with the more complicated side and transform it in a step-by-step fashion until it looks exactly like the other side.
- Analyze the identity and look for opportunities to apply the fundamental identities.
- Try using one or more of the following techniques:
 1. Rewrite the more complicated side in terms of sines and cosines.
 2. Factor out the greatest common factor.
 3. Separate a single-term quotient into two terms:
$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad \text{and} \quad \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}.$$
 4. Combine fractional expressions using the least common denominator.
 5. Multiply the numerator and the denominator by a binomial factor that appears on the other side of the identity.
- Don't be afraid to stop and start over again if you are not getting anywhere. Creative puzzle solvers know that strategies leading to dead ends often provide good problem-solving ideas.