**Section 5.2** (Sum and Difference Formulas)

Cosine of the difference of two angles (see pg. 597 and below steps for visual proof or explore on board with time)

* + Start with unit circle and angles α and β having endpoints P and Q
	+ List coordinates of P and Q for these angles (using cos α for x and sin α for y)
	+ Connect P and Q with line segment and apply distance formula
	+ Rotate triangle in line with x-axis and maintaining angle α – β
	+ Re-apply distance formula to get equivalent distance between P and Q (one point should now be (1,0))

**cos (α – β) = cos α cos β + sin α sin β**

Example: Find the exact value of cos 30o using only 90o and 60o angles

Example: Find the exact value of cos 80o cos 20o + sin 80o sin 20o

Example: Verify the identity $\frac{\cos((α-β))}{\cos(α\cos(β))}$ = 1 + tan α tan β

We can use this difference formula to verify and establish other identities (see cofunction complements discussion pg. 599 that shows cos(π/2 – θ) = sin θ)

**cos (α + β) = cos α cos β – sin α sin β**

**cos (α – β) = cos α cos β + sin α sin β**

**sin (α + β) = sin α cos β + cos α sin β**

**sin (α – β) = sin α cos β – cos α sin β**

Example: Suppose that sin α = 4/5 for a quadrant II angle α and sin β = ½ for a quadrant I angle β. Find the exact value of the following…

 cos α cos β cos (α + β) sin (α + β)

If we rewrite tan (α + β) using sin and cos, we can develop a formula for the tangent of sums and differences

Example: Derive the identity for tan (α + β) -- Hint: divide top and bottom by cos α cos β

(Optional) Example: Derive the identity for tan (α – β) -- Hint: use tan (α – β) = tan [α +(– β)] (and tan is odd)

(Optional) Example: Verify the identity tan (x + π) = tan x

**tan (α + β) =** $\frac{tan α + tan β}{1 - tan α tan β}$ **tan (α – β) =** $\frac{tan α - tan β}{1 + tan α tan β}$

Example (Extra Credit?): (#80 – pg. 606) A tuning fork is held a certain distance from your ears and struck. Your eardrums’ vibrations after t seconds are given by p = 3 sin (2t). When a second tuning fork is struck, the formula

p = 2 sin (2t + π) describes the effects of the sound on the eardrums’ vibrations. The total vibrations then are given by p = 3 sin (2t) + 2 sin (2t + π).

* + Simplify p to s single term containing the sin
	+ If the amplitude of p is zero, no sound is heard. Based on your equation in the part above, does this occur with the two tuning forks in the exercise (Explain)