**Section 5.3** (Double-Angle, Power-Reducing, and Half-Angle Formulas)

In this section, we continue to use sum formulas to look at other basic identities

*Double-Angle Formulas*

**sin 2θ** = sin (θ + θ) = **= 2 sin θ cos θ**

**cos 2θ** = cos (θ + θ) = **= cos2θ – sin2θ**

**tan 2θ** = tan (θ + θ) = **=** $\frac{2 tan θ}{1 - tan^{2} θ}$

Example: If sin θ = 4/5 and θ lies in quadrant II, find the exact value of the following…



 sin 2θ cos 2θ tan 2θ

Example: Find the exact value of cos2 15o – sin2 15o

There are three forms of the double angle formula for cos 2θ. Using the Pythagorean identity, sin2 θ + cos2 θ = 1, we can derive (sometimes it is helpful to express in terms of just one trigonometric function) …

 **cos 2θ =** cos2 θ – sin2 θ = cos2 θ – (1 – cos2 θ) = cos2 θ – 1 + cos2 θ **= 2 cos2 θ – 1**

 **cos 2θ =** cos2 θ – sin2 θ = **= 1 – 2 sin2 θ**

(Optional) Example: Verify the identity sin 3θ = 3 sin θ – 4 sin3 θ

*Power-Reducing Formulas*

cos 2θ = 1 – 2 sin2 θ => => **sin2 θ =** $\frac{1-cos 2θ}{2}$

(-- solve for sin2 θ --)

cos 2θ = 2 cos2 θ – 1 => => **cos2 θ =** $\frac{1+cos 2θ}{2}$

**tan2 θ** = $\frac{ sin^{2} θ}{cos^{2} θ}$ = **=** $\frac{1-cos 2θ}{1+ cos 2θ}$

Example: Write an equivalent expression for sin4 x that doesn’t contain powers of trig. functions greater than 1

*Half-Angle Formulas*

**sin** $\frac{α}{2}$ **= ±** $\sqrt{\frac{1-cos α}{2}}$ **cos** $\frac{α}{2}$ **= ±** $\sqrt{\frac{1+cos α}{2}}$ **tan** $\frac{α}{2}$ **= ±** $\sqrt{\frac{1-cos α}{1+cos α}}$

(The ± symbol indicates that you determine the sign based on where the half-angle lies)

Example: Use cos 210o = – $\sqrt{3}$ / 2 to find the exact value of cos 105o

Some alternate forms of tan (α/2) don’t require us to determine sign

Example: See example 6 (pg. 613) and verify the identity tan θ = $\frac{sin 2θ}{1+\cos(2θ)}$

These examples lead to the other forms of the tangent half-angle formula

**tan** $\frac{α}{2}$ **=** $\frac{1-cos α}{sin α}$ **=** $\frac{sin α}{1+cos α}$

(Optional) Example: Verify the identity tan $\frac{α}{2}$ = $\frac{sec α}{sec α csc α+csc α}$

SEE SUMMARY OF PRINCIPAL TRIGONOMETRIC IDENTITIES ON PG. 614