Section 5.3 (Double-Angle, Power-Reducing, and Half-Angle Formulas)

In this section, we continue to use sum formulas to look at other basic identities

Double-Angle Formulas

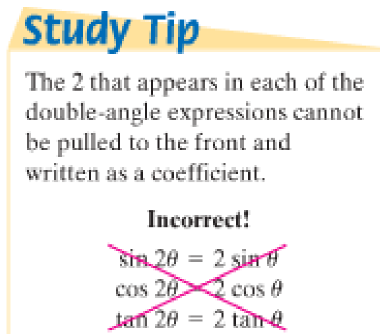
sin 2θ = sin (θ + θ) = **= 2 sin θ cos θ**

cos 2θ = cos (θ + θ) = **= $\cos^2\theta - \sin^2\theta$**

tan 2θ = tan (θ + θ) = $= \dfrac{2\tan\theta}{1-\tan^2\theta}$

Example: If sin θ = 4/5 and θ lies in quadrant II, find the exact value of the following…

sin 2θ   cos 2θ   tan 2θ


Study Tip

The 2 that appears in each of the double-angle expressions cannot be pulled to the front and written as a coefficient.

Incorrect!

$\sin 2\theta = 2\sin\theta$

$\cos 2\theta = 2\cos\theta$

$\tan 2\theta = 2\tan\theta$

Example: Find the exact value of $\cos^2 15^\circ - \sin^2 15^\circ$

There are three forms of the double angle formula for cos 2θ. Using the Pythagorean identity, $\sin^2\theta + \cos^2\theta = 1$, we can derive (sometimes it is helpful to express in terms of just one trigonometric function) …

cos 2θ = $\cos^2\theta - \sin^2\theta = \cos^2\theta - (1-\cos^2\theta) = \cos^2\theta - 1 + \cos^2\theta$ **$= 2\cos^2\theta - 1$**

cos 2θ = $\cos^2\theta - \sin^2\theta =$ **$= 1 - 2\sin^2\theta$**

(Optional) Example: Verify the identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

Power-Reducing Formulas

$\cos 2\theta = 1 - 2\sin^2\theta$ => => $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

(-- solve for $\sin^2\theta$ --)

$\cos 2\theta = 2\cos^2\theta - 1$ => => $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$\tan^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta} =$ $= \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$

Book problems:

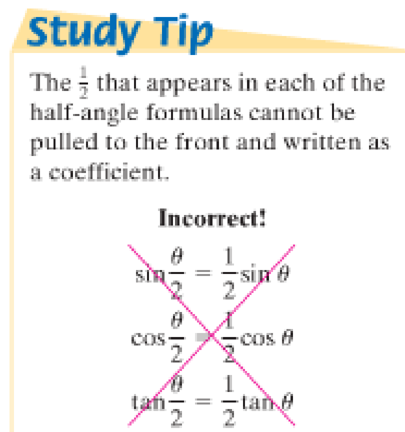
Example: Write an equivalent expression for $\sin^4 x$ that doesn't contain powers of trig. functions greater than 1

Half-Angle Formulas

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}} \qquad \tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

(The ± symbol indicates that you determine the sign based on where the half-angle lies)

Example: Use $\cos 210^{o} = -\sqrt{3}/2$ to find the exact value of $\cos 105^{o}$


Study Tip

The $\frac{1}{2}$ that appears in each of the half-angle formulas cannot be pulled to the front and written as a coefficient.

Incorrect!

$$\sin\frac{\theta}{2} = \frac{1}{2}\sin\theta$$
$$\cos\frac{\theta}{2} = \frac{1}{2}\cos\theta$$
$$\tan\frac{\theta}{2} = \frac{1}{2}\tan\theta$$

Some alternate forms of tan (α/2) don't require us to determine sign

Example: See example 6 (pg. 613) and verify the identity $\tan\theta = \frac{\sin 2\theta}{1+\cos 2\theta}$

These examples lead to the other forms of the tangent half-angle formula

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$$

(Optional) Example: Verify the identity $\tan\frac{\alpha}{2} = \frac{\sec\alpha}{\sec\alpha\csc\alpha+\csc\alpha}$

SEE SUMMARY OF PRINCIPAL TRIGONOMETRIC IDENTITIES ON PG. 614

Book problems: