**Section 5.4** (Product-to-Sum and Sum-to-Product Formulas)

We can write products of sin and / or cos functions as sums or differences…

*Product-to-Sum Formulas*

**sin α sin β = ½ [cos (α – β) – cos (α + β)]**

**cos α cos β = ½ [cos (α – β) + cos (α + β)]**

**sin α cos β = ½ [sin (α + β) + sin (α – β)]**

**cos α sin β = ½ [sin (α + β) – sin (α – β)]**

We can derive these formulas using the difference and sum formulas of cos (and sin) – we’ll look at the 2nd formula.

Add difference and sum formulas for cos => cos (α – β) = cos α cos β + sin α sin β

 + cos (α + β) = cos α cos β – sin α sin β

 cos (α – β) + cos (α + β) = 2 cos α cos β …

Example: Express each of the following products as a sum or difference…

 sin 5x sin 2x cos 7x cos x

*Sum-to-Product Formulas*

**sin α + sin β = 2 sin** $\left(\frac{α+ β}{2}\right)$ **cos** $\left(\frac{α- β}{2}\right)$

**sin α – sin β = 2 sin** $\left(\frac{α- β}{2}\right)$ **cos** $\left(\frac{α+ β}{2}\right)$

**cos α + cos β = 2 cos** $\left(\frac{α+ β}{2}\right)$ **cos** $\left(\frac{α- β}{2}\right)$

**cos α – cos β = – 2 sin** $\left(\frac{α+ β}{2}\right)$ **sin** $\left(\frac{α- β}{2}\right)$

We can derive these formulas using the Product-to-Sum Formulas above and going in reverse (see pg. 621)

Consider the right side from the 2nd formula above 2 sin $\left(\frac{α- β}{2}\right)$ cos $\left(\frac{α+ β}{2}\right)$

 -- apply product-to-sum 2 \* **½** [sin $\left(\frac{α- β}{2}\right.$ + $\left.\frac{α+ β}{2}\right)$ + sin $\left(\frac{α- β}{2}\right.$ – $\left.\frac{α+ β}{2}\right)$ ]

 -- simplify…

Example: Express each sum as a product…

 sin 7x + sin 3x cos 3x + cos 2x

Applying these identities in the numerator and denominator is often helpful in verifying identities that contain a fraction on one side with sums and differences of sin and/or cos

(Optional) Example: Verify the identity $\frac{\cos(3x) –\cos(x)}{\sin(3x+\sin(x))}$ = – tan x

**Review** **online HW** and Examine #45 on pg. 624 (as time permits)