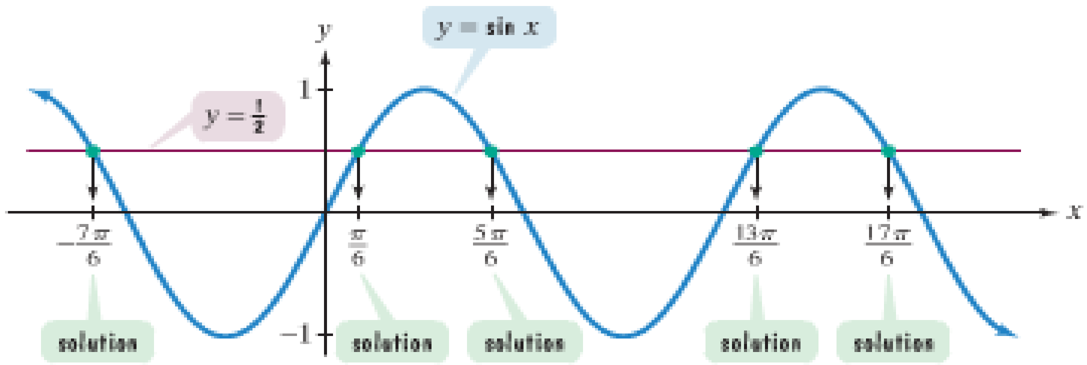
Section 5.5 (Trigonometric Equations)

A trigonometric equation is simply an equation that contains a trigonometric expression with a variable. Consider the equation sin x = ½. One solution is π/6 because sin π/6 = ½ (how do we know this). Is that the only solution? What other solutions can be found in one period of the sin [0,2π)? How can you represent the solutions?



The method for solving trig. functions is similar to that for solving for variables in general.

- Isolate trig. function on one side of equation
- Solve for the variable

Example: Solve the equation 5 sin x = 3 sin x + $\sqrt{3}$

Keeping in mind that these equations can involve multiple solutions, often we will restrict our ourselves 0 ≤ x < 2π

Example: Solve the following equations…

tan x = $\sqrt{3}$, 0 ≤ x < 2π

(challenge if time) sin 2x = $\frac{1}{2}$, 0 ≤ x < 2π

So far, we have looked at cases where solutions were found by knowing exact values. Often they do not contain these known values and we need to rely on calculator ($\sin^{-1}$, $\cos^{-1}$, etc.) functions. Review…

Examples: Solve each equation for 0 ≤ x < 2π (3 decimal places)

tan x = 3.1044

sin x = – 0.2315

Book problems:

<u>OPTIONAL MATERIAL</u> (only cover if time allows, otherwise, do HW problems)

Sometimes these trigonometric expressions occur in quadratic form. Again this is similar to quadratic equations containing only variables (try factoring followed by quadratic formula or square root property)

<u>Example</u>: Solve the following equations (and review quadratic formula with time if needed)

$x^2 + 2x - 15 = 0$ $2 \sin^2 x - 3 \sin x + 1 = 0$, $0 \le x < 2\pi$

$4x^2 - 5x - 6 = 0$ $4 \cos^2 x - 3 = 0$, $0 \le x < 2\pi$

What if the equation contains more than one trigonometric function?

<u>Example</u>: Solve the equation $\sin x \tan x = \sin x$, $0 \le x < 2\pi$

Move all terms to one side to get 0 on the other side …

Use factoring to separate the functions …

This technique does not always work if we can't separate the functions via factoring. Often trig. identities can help

<u>Example</u>: Solve the following equations

$2 \sin^2 x - 3 \cos x = 0$, $0 \le x < 2\pi$ $\cos 2x + \sin x = 0$, $0 \le x < 2\pi$

<u>Example</u>: Solve the equation $\sin x \cos x = -\frac{1}{2}$, $0 \le x < 2\pi$

Hint: change both sides of the equation into an expression for which we can use an identity

Often we rely on the identity $\sin^2 x + \cos^2 x = 1$, so we will square both sides of the equation in order to substitute

(Note: When raising both sides of an equation to an even power, you must check each proposed solution: $x = 2$ for example)

<u>Example</u>: Solve the equation $\cos x - \sin x = -1$, $0 \le x < 2\pi$

Book problems: