Section 6.2 (The Law of Cosines)

In the last section, we examined a couple of cases involving oblique triangles. In this section, we continue that discussion with 2 more cases

1. SAS (Side-Angle-Side) – two sides and the included angle are known
2. SSS (Side-Side-Side) – three sides are known

Given a triangle with angles A,B,C, and opposite sides a,b,c, then the square of one side of the triangle equals the sum of the squares of the other 2 sides minus twice their product times the cos of their included angle

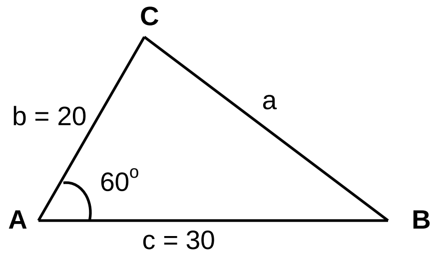
$$\mathbf{a^2 = b^2 + c^2 - 2(bc)\cos A \qquad b^2 = a^2 + c^2 + - 2(ac)\cos B \qquad c^2 = a^2 + b^2 + - 2(ab)\cos C}$$

See proof on page 657 involving triangle ABC within a rectangular coordinate system (and dropping a vertical line to create right triangles)

Within an SAS triangle, none of the ratios used in the Law of Sines can be known, so we first utilize the Law of Cosines to find the length of a side opposite the given angle. **Solving a SAS triangle** involves…

- Use Law of Cosines to find the length of the side opposite the given angle
- Use Law of Sines (and the fact that the sum of the angles is 180) to complete triangle

Example: Solve the given triangle



Solving an SSS triangle involves…

- Use Law of Cosines to find the angle **opposite the longest side**
- Use Law of Sines (and the fact that the sum of the angles is 180) to complete triangle

Example: Sketch and solve a triangle ABC if a = 8, b = 10, and c = 5 (round angle to nearest degree)

Example: Two airplanes leave the Church Hill Airport (CHA) at the same time on different runways. One flies directly north (towards Detroit) at 400 miles per hour. The other flies on a bearing of N75^{o}E (towards Buffalo) at 350 miles per hour. How far apart will the airplanes be after 2 hours?

Book problems:

Example: After the Ala-barf-a loss to Tennessee, you have decided to just take a boat and get out of town. So you drive to Jacksonville and leave the pier heading directly east (on a boat– probably the Hokie Boat Cruise Lines). After traveling for 25 miles, there is a report of rough seas directly south and up ahead. The captain turns the boat and follows a bearing of S40^{o}W for 13.5 miles. At this time, how far are you from the boat's pier? What bearing could the boat have originally taken to arrive at this new spot? (see figure on bottom of pg. 662 for more details)

Many years ago, Greek mathematician Heron of Alexandria derived a formula for the area of a triangle in terms of the lengths of its sides (a more modern derivation uses the Law of Cosines and can be found in the book appendix)

Heron's Formula for the Area of a Triangle

The area of a triangle with sides a, b, and c is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is one-half its perimeter: $s = \frac{1}{2}(a+b+c)$.

Use Heron's formula in the following example and examine online HW exercises with time…

Example: Find the area of the triangle with a = 6 m., b = 16 m., and c = 18 m. (round to nearest square meter)

Is this how triangle areas are currently found? Test the formula against right triangles (A = ½bh). Excel??

Book problems: