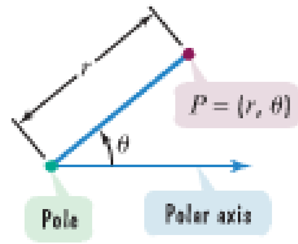
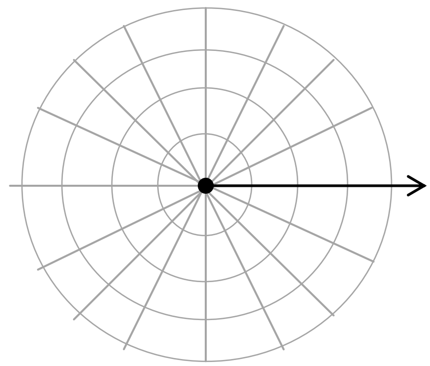
Section 6.3 (Polar Coordinates)

In order to better define and study circular and orbital motion, the polar coordinate system is often utilized (navigation, gravitational fields, radio antennas as well). The polar coordinate system is represented by a pole (like the origin) and a horizontal ray extending to the right (polar axis). A point P is represented by an ordered pair **(r, θ)** where r is the distance from the pole to P and θ is the angle from the polar axis to P. Positive angles are measured counter-clockwise (negative angles clockwise).



Point P is located |r| units from the pole. If r > 0 => point lies along θ, r < 0 => point lies along ray opposite θ, r = 0 => point lies at the pole.

Example: Plot the points with the following coordinates **A (3, 315°)** **B (–2, π)** **C (–1, –π/2)**



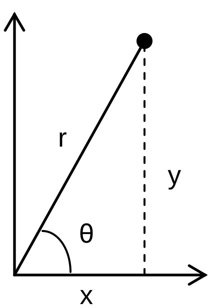
Unlike rectangular coordinates, where points only have one representation, points in polar coordinates can be represented in many different ways, including the following…

(r, θ) = (r, θ + 2π) and **(r, θ) = (–r, θ + π)**

(r, θ + 2nπ) (–r, θ + π + 2nπ)

Example: Find another representation of (5, π/4) in which…

r is positive and 2π < θ < 4π r is negative and 0 < θ < 2π r is positive and – 2π < θ < 0



Points in polar coordinates can also be expressed in rectangular coordinates (see pg. 667 for more details). Notice the point in both coordinate systems here. And consider the relationships…

$x^2 + y^2 = r^2$ sin θ = cos θ = tan θ =

Solving these equations to better represent x and y in relation to r and θ, we get

$\mathbf{x^2 + y^2 = r^2}$ **x =r cos θ** **y = r sin θ** **tan θ = y / x**

To convert from polar to rectangular, we will use what 2 formulas?

Example: Find the rectangular coordinates of (3, π) and (–10, π/6)

Remembering that there are infinitely many polar representations, we can also convert from rectangular to polar

- assume r is positive and θ is positive and less than 2π –

Use $x^2 + y^2 = r^2$ or $\mathbf{r} = \sqrt{x^2+y^2}$ and **tan θ = y / x** (remember angle restrictions)

Book problems:

Example: Find polar coordinates of the point whose rectangular coordinates are $(1, -\sqrt{3})$ and $(0, -4)$

A polar equation is an equation whose variables are r and θ. We can convert rectangular equations to polar equations using $x = r\cos\theta$ and $y = r\sin\theta$…

Example: Convert each rectangular equation to a polar equation that expresses r in terms of θ (solved for r)

$3x - y = 6$ $\qquad$ $x^2 + (y + 1)^2 = 1$

Converting from a polar equation to rectangular equation also uses the polar equations seen, but many times requires us to square both sides, take the tangent of both sides, or multiply r on both sides (takes practice)…

Example: Convert each polar equation to a rectangular equation in x and y (sketch graph with time)

$r = 4$ (we need r^2 to use our formula, so what should we do)

$\theta = \frac{3\pi}{4}$ (equation has tan θ, so what can we do here)

$r = -2\sec\theta$ (equations only have $r\cos\theta$ and $r\sin\theta$, so…)

$r = 10\sin\theta$

Book problems: