**Section 6.5** (Complex Numbers in Polar Form: DeMoivre’s Theorem)

A complex number z = a + bi can be represented as a point (a,b) in a coordinate plane, as shown in the figure

This coordinate system is called the complex plane and every complex number corresponds to a number in this plane

b

Imaginary Axis

Real Axis

*O*

a

**z = a + bi**

Example: Plot each number in the complex plane:

z = 2 + 3i z = – 3 – 5i z = – 4 z = – i

The absolute value of a complex number is the distance from the origin to that point in the complex plane (sketch line / triangle in figure to examine the distance from (a,b) to the origin)



Example: Determine the absolute value of each of z = 5 + 12i and 2 – 3i

A complex number of the form z = a + bi is said to be in rectangular form. Looking back at our figure, and letting r represent the distance (side) from point (a,b) to the origin, we have

Imaginary Axis

Real Axis

*O*

a

r

b

(a,b)

 **cos () =** **sin () =** **tan () =**

 **a = r cos(θ) b = r sin(θ)**

Substituting back into rectangular form gives

 z = a + bi = r cos(θ) + (r sin(θ))i = **z =** **r (cos(θ) + i sin(θ))**

Example: Plot **z = –1 – i**$\sqrt{3}$ in the complex plane and then express z in polar form (radians for angle measure)

To multiply 2 complex numbers in polar form, we multiply the moduli (r1\*r2) and add the arguments (cos(θ1 + θ2) + i sin(θ1 + θ2)) – see explanation and proof on pg. 690 in book

**z1z2 = r1r2[cos(θ1 + θ2) + i sin(θ1 + θ2)]**

Example: Find the product of the complex numbers given (leave answer in polar form)

 **z1 = 6(cos(40o) + i sin(40o)) z2 = 5(cos(20o) + i sin(20o))**

To divide 2 complex numbers in polar form, we divide the moduli (r1/r2) and subtract the arguments (cos(θ1 + θ2) + i sin(θ1 + θ2)) – see proof using algebraic complex number division in appendix in book if interested

Example: Find the quotient of the complex numbers given (leave answer in polar form)

 **z1 = 50(cos**$\left(\frac{4π}{3}\right)$ **+ i sin**$\left(\frac{4π}{3}\right)$**) z2 = 5(cos**$\left(\frac{π}{3}\right)$**)+ i sin**$\left(\frac{π}{3}\right)$**))**

We can also find powers of complex numbers in polar form using DeMoivre’s Theorem. Consider z2 …

 z = r (cos(θ) + i sin(θ)) z2 = r (cos(θ) + i sin(θ)) r (cos(θ) + i sin(θ)) = r2 (cos(2θ) + i sin(2θ))...

This pattern continues for higher powers of z in polar form meaning **zn = rn (cos(nθ) + i sin(nθ))**

Example: Find **[2 (cos(30o) + i sin(30o))]5** and write the answer in rectangular form

Example: Find **(1 + i)4** using DeMoivre’s Theorem and write the answer in rectangular form

OPTIONAL ---

DeMoivre’s Theorem can basically be reverse engineered to find complex roots as well (see in-depth discussion on pg. 693 in the book)...

 Complex number w = r (cos(θ) + i sin(θ)) has n distinct nth roots that can be given as

 **zk =** $\sqrt[n]{r}\left[cos\left(\frac{θ+2πk}{n}\right)+i sin\left(\frac{θ+2πk}{n}\right)\right]$ **zk =** $\sqrt[n]{r}\left[cos\left(\frac{θ+360^{o}k}{n}\right)+i sin\left(\frac{θ+360^{o}k}{n}\right)\right]$

 where k = 0, 1, 2,…,n – 1 (note that each root has the same modulus)

Example: Find all the complex fourth roots of **16(cos(60o) + i sin(60o))** and write the roots in polar form

Example: Find all the cube roots of 27. Write roots in rectangular form…