## Section 9.3 (The Parabola)

Parabolas are explored in Pre-Calculus Algebra (having the form $y=a(x-h)^{2}+k$ or $y=a x^{2}+b x+c$ ), so we will look at a couple different attributes in this class

A parabola is the set of all points in a plane that are equidistant from a fixed line (directrix) and a fixed point (focus) not on the line


Here is a summary of what you should already know about graphing parabolas.

$$
\text { Graphing } y=a(x-h)^{2}+k \text { and } y=a x^{2}+b x+c
$$

1. If $a>0$, the graph opens upward. If $a<0$, the graph opens downward.
2. The vertex of $y=a(x-h)^{2}+k$ is $(h, k)$.
3. The $x$-coordinate of the vertex of $y=a x^{2}+b x+c$ is $x=-\frac{b}{2 a}$.



Measuring the distances to a point on a parabola from the focus ( $p, 0$ ) and directrix ( $-\mathrm{p}, \mathrm{y}$ ) as $\mathrm{d}_{1}=d_{2}$, we can use distance formulas to derive another form for parabolas with a vertex at the origin (see derivation on pgs. 901-902) as $\mathbf{y}^{2}=4 p x$ (opens right/left with focus on $x$-axis of symmetry) or $\mathbf{x}^{2}=4 p y$ (opens up/down with focus on $y$-axis)

Example: Find the focus and directrix of the parabola given by the following equations and graph using points above/below or left/right of the focus (notice that these points are $+/-2 p$ from the focus)

$$
y^{2}=8 x \quad x^{2}=-12 y
$$

$4 p=$ $\qquad$ ...


Example: Find the standard form of the equation of a parabola with focus $(\mathbf{8}, \mathbf{0})$ and directrix $\mathbf{x}=\mathbf{- 8}$

Again, not all parabolas are centered at the origin and may be translated (techniques remain the same, but vertices, foci, directrix are now in relation to the new center point) - see figures in book...

| Equation | Vertex | Axis of Symmetry | Focus | Directrix | Description |
| :---: | :--- | :--- | :--- | :--- | :--- |
| $(y-k)^{2}=4 p(x-h)$ | $(h, k)$ | Horizontal | $(h+p, k)$ | $x=h-p$ | If $p>0$, opens to the right. <br> If $p<0$, opens to the left. |
| $(x-h)^{2}=4 p(y-k)$ | $(h, k)$ | Vertical | $(h, k+p)$ | $y=k-p$ | If $p>0$, opens upward. <br> If $p<0$, opens downward. |

Example: Find the vertex, focus, and directrix of the following (and graph if time)

$$
(x-2)^{2}=4(y+1) \quad y^{2}+2 y+4 x-7=0
$$

There are many applications of parabolas including arches/cables for bridges, solar cookers, reflectors for lights (flashlights), satellite dishes, etc.

Example: Cookie Monster is making an effort to go green, so he has decided to make a solar cooker to help him bake cookies (satellite dish basically) with a diameter of 6 feet and a depth of 1 foot. Where should he put the cooker (focus) to get the maximum reflected sun rays to bake the cookies?

