<u>Sections 9.4 and 9.6 Summary</u> <u>(Rotation of Axes / Conic Sections in Polar Coordinates)</u>

Section 9.4 (Rotation of Axes)

Given a conic equation ($Ax^2 + Cy^2 + Dx + Ey + F = 0$) not in standard form such as ., we can determine which conic section we have by examining A, C, and AC (coefficient of x^2, y^2)…

A = C, circle   AC = 0, parabola   AC > 0, ellipse   AC < 0, hyperbola

<u>Example</u>: Identify the graph of each of the following

$3x^2 + 2y^2 + 12x – 4y + 2 = 0$

$x^2 + y^2 – 6x + y + 3 = 0$

$y^2 – 12x – 4y + 52 = 0$

$9x^2 – 16y^2 – 90x + 64y + 17 = 0$

The presence of an xy-term in conic equations sometimes rotates the conic over a given angle

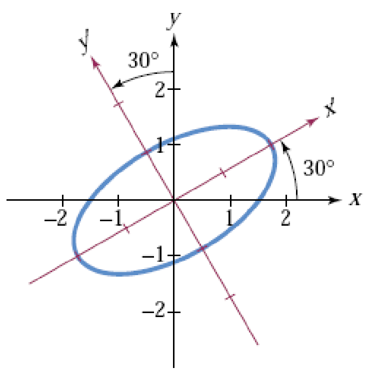

Figure 9.46 The graph of $7x^2 - 6\sqrt{3}\,xy + 13y^2 - 16 = 0$, a rotated ellipse

See page 916 and Example 2 in the book (and Ex. 3 result on pg. 920)

Writing the Equation of a Rotated Conic in Standard Form

1. Use the given equation
$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,\ B \neq 0$$
to find $\cot 2\theta$.
$$\cot 2\theta = \frac{A - C}{B}$$
2. Use the expression for $\cot 2\theta$ to determine θ, the angle of rotation.
3. Substitute θ in the rotation formulas
$$x = x'\cos\theta - y'\sin\theta \quad \text{and} \quad y = x'\sin\theta + y'\cos\theta$$
and simplify.
4. Substitute the expressions for x and y from the rotation formulas in the given equation and simplify. The resulting equation should have no $x'y'$-term.
5. Write the equation involving x' and y' in standard form.

Section 9.6 (Conic Sections in Polar Coordinates)

Another way to represent conic sections is through polar equations (remember r, θ) using eccentricity

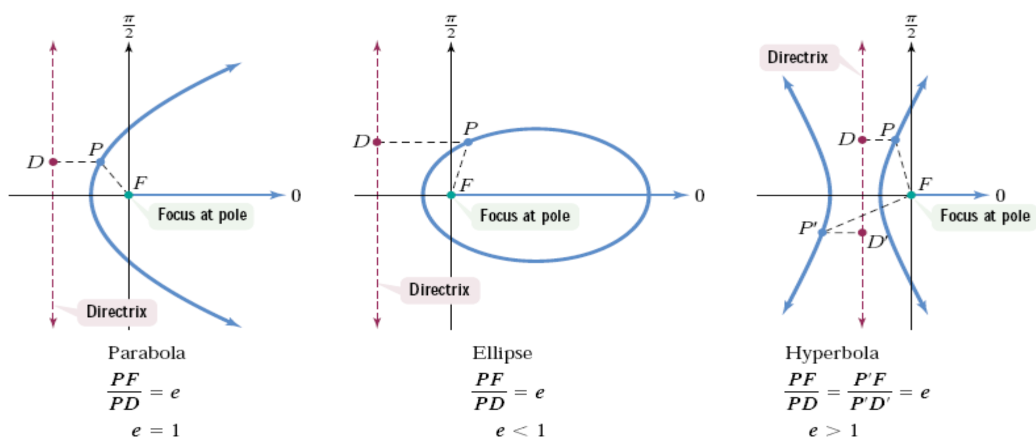

Figure 9.59 The eccentricity for each conic

<u>Equation Forms</u> :

r = $\dfrac{ep}{1 +/- e\cos\theta}$

r = $\dfrac{ep}{1 +/- e\sin\theta}$

e = eccentricity

p = dist. from focus to directrix

SEE BOOK EXAMPLES FOR MORE