**Section 9.5** (Parametric Equations)

 In a recent game against the Barney’s, Ernie threw a baseball from an initial height of 4 feet, with an initial velocity of 90 feet per second and at an angle of 40o with the horizontal. After t seconds the location of the ball can be described by…

 x = (90 cos 40o)t - ball’s horizontal distance in feet - - AND -

 y = 4 + (90 sin 40o)t – 16t2 - ball’s vertical distance in feet –

Using these equations as time (t) changes, we can calculate the location of the ball at any time.

This set of equations (x and y as functions of t) are called **parametric equations**

|  |  |  |
| --- | --- | --- |
| Time (t) | x(t) (horizontal) | y(t) (vertical) |
| 0 | 0 ft. | 4 ft. |
| 1 | 90 cos 40o ~ 69 ft. | ~ 46 ft. |
| 2 | ~ 138 ft. | ~64 ft. |
| 3 | ~ 207 ft. | ~34 ft. |
| 3.6 | ~248 ft. | ~0 ft. |

50

100

150

200

250

20

40

60

80

t = 1

t = 2

t = 3

A **plane curve** is the set of ordered pairs (x,y) where x = f(t) and y = g(t) for t in an interval I and the equations x = f(t) and y = g(t) are called **parametric equations** for the curve

Example: Graph the plane curve defined by the parametric equations **x = t2 + 1** , **y = 3t** , **-2 ≤ t ≤ 2**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |



We examined a variety of points in the first example to graph a plane curve. Another method is to eliminate the parameter (t) to write one equation in x and y equivalent to the two equations (see discussion on pg. 927).

 1. Solve for t in one of the parametric equations 2. Substitute the expression for t in the other equation

(you may need to change the domain of the rectangular equation to be consistent with the domain for the parametric equation)

Example: Sketch the plane curve represented by the following parametric equations by eliminating the parameter

 (use identity sin2 t + cos2 t = 1)

 **x =** $\sqrt{t}$ **y = 2t – 1 x = 6 cos t y = 4 sin t π ≤ t ≤ 2π**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |
|  |  |   |   |   |   |   |   |   |   |



It is sometimes helpful in applications to find a set of parametric equations for a given function (there are often multiple possible sets of parametric equations for a given function, but remember that the parametric equations must allow x to take on all the values in the domain of the original equation. One method if y = f(x) is to let x = t and go from there…

Example: Find a set of parametric equations for the parabola whose equation is **y = x2 – 25**

See cycloid application in the book for details on a different application problem, but let’s explore our first example a bit more.

The path of a projectile launched h feet above the ground with an initial velocity of v0 ft. per second and at an angle θ with the horizontal is given by

 x = (v0 cos θ)t - object’s horizontal distance in feet - - AND -

 y = h + (v0 sin θ)t – 16t2 - object’s vertical distance in feet –

Example: A trigonometry book is launched by an angry student with an initial velocity of 200 ft. per second at an angle of 42o with the horizontal. The book was launched from a height of 4 feet.

1. Find the parametric equations that describe the position of the book as a function of time.
2. Describe the book’s position after t = 1 second and after t = 3 seconds
3. How long (to the nearest tenth of a sec.) is the book in flight, and how far does the book fly (horizontally)?