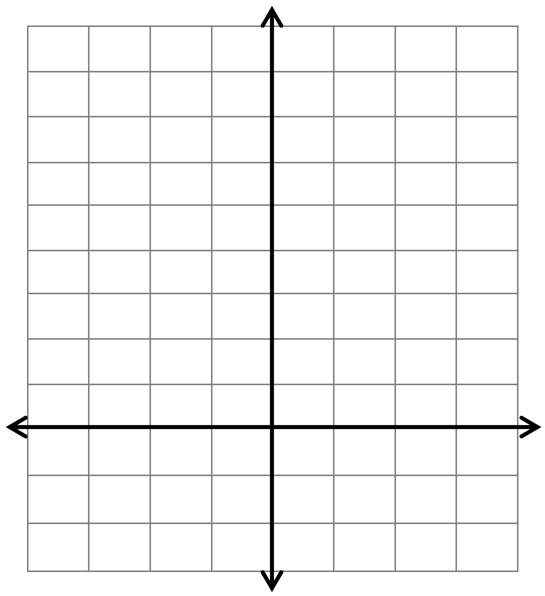
## **Section 1.2** (Finding Limits Graphically and Numerically)

Review section 1.1 for an introduction to calculus (mathematics of change) and list 2 classic problems in calculus

1. The ________________________________________ problem
2. The ________________________________________ problem

Sketch the graph of **f(x) =** $\frac{x^2-1}{x-1}$ , x ≠ 1 on the given axes and observe the behavior of the graph



| x | f(x) = $\frac{x^2-1}{x-1}$ |
|---|---|
| **– 1** | |
| **0** | |
| **0.5** | |
| **0.9** | |
| **1.01** | |
| **2** | |

Though our function is not defined at x = 1, we can observe that as our x-values approach 1 from the left (0.5,0.9,0.999,…) and from the right (1.5,1.1,1.0001,…), our function f(x) approaches__________.

The limit of f(x) as x approaches c is given by the equation **L =** $\lim_{x\to c} f(x)$, or in the given example …

$$\lim_{x\to 1} \frac{x^2-1}{x-1} =$$

Examples: Find the limits of the functions given below…

$\lim_{x\to 1} \frac{x^2+3}{x+1}$ $\qquad\qquad$ $\lim_{x\to 2} |x-3|$ $\qquad\qquad$ $\lim_{x\to 0} \frac{sin(x)}{x}$

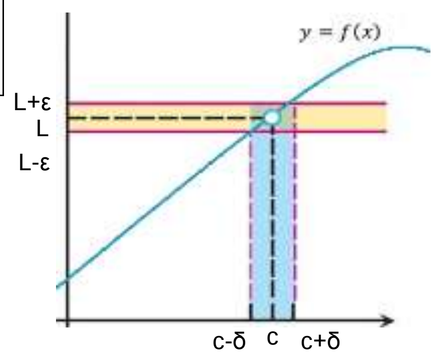
Homework problems: 1,4,9,15,18,20,25 (required) 39,43,61 (recommended)

Some limits fail to exist. Consider the results when examining…

$$\lim_{x\to 1}\frac{|x-1|}{x-1} \qquad\qquad \lim_{x\to 0}\frac{2}{x^4} \qquad\qquad \lim_{x\to 0} cos\left(\frac{1}{x}\right)\ldots$$

| **Definition of limit** |
|---|
| Let *f* be a function defined on an open interval containing *c* and let *L* be a real number. The statement $\lim_{x\to c} f(x) = L$ means that for each ε > 0, there exists a δ > 0 such that if **0 < \|*x* – *c*\| < δ,** then **\|*f(x)* – *L*\| < ε** |

For a given +/- change in y < ε, there is a corresponding change in x < δ




The formal definition of limits can be used to <u>prove</u> limits of functions

Deep-thinking exercises

<u>Example</u>: Find the limit L. Then find δ > 0 such that |f(x) – L| < 0.01 whenever 0 < |x – c| < δ

$$\lim_{x\to 0}\left(4-\frac{x}{2}\right) =$$

whenever 0 < |x –       | < δ

<u>Example</u>: Find the limit L. Then use the ε – δ definition to prove that the limit is L

$$\lim_{x\to -3}(2x+5)$$

<u>Application Example</u>: Bert and Ernie created a sporting goods manufacturer for Sesame Street and designed a golf ball having a volume of 2.48 cubic inches.

(a) What is the radius of the golf ball?

(b) If the ball's volume can vary between 2.45 and 2.51 cubic inches, how can the radius vary?

(c) Consider the ε – δ definition of limit to describe this situation (what are ε and δ)…

Homework problems: 1,4,9,15,18,20,25 (required) 39,43,61 (recommended)