**Section 1.3** (Evaluating Limits Analytically)

In section 1.2, we introduced the concept of limits and learned that the limit of a function f(x) as x approaches c (a constant) isn’t necessarily the function value at x = c, but oftentimes this is the case. If a function f(x) is **continuous at c**, then . Here are some basic limits where b and c are real numbers and n is a positive integer…

( What do each of these look like graphically? )

Furthermore, with and …

1. Scalar multiple:
2. Sum or difference:
3. Product:
4. Quotient: (provided K ≠ 0)
5. Power:

Examples: Evaluate the following limits…

From the last example above, notice that the limit is the same as the function evaluated at x = – 2 . This simple substitution property is valid for all polynomials and rational functions with nonzero denominators. Consider polynomial functions p(x) and q(x) and rational function r(x) …

 (where q(c) ≠ 0)

Examples: Evaluate the following limits…

Similar rules apply for roots and composite functions …

(valid for c>0 if n even)

Example: Evaluate the following limit…

 => g(x) = x2 + 9, f(x) = => => =>

 Using direct substitution, we also have the trigonometric functions (assume c is in the domain of the function)…

 …

Examples: Evaluate the following limits…

Review factoring of **f(x) = x3 – 1** and **g(x) = x3 + 1** and discuss example 6 in the book…

 **f(x) = x3 – 1 = (x – 1)(x2 + x + 1) g(x) = x3 + 1 =**

Theorem 1.7: If a function f(x) = g(x) for all x ≠ c and the limit of g(x) as x -> c exists, then

Examples: Divide out factors and / or rationalize to find the following limits…

Review the squeeze theorem (sandwich, pinching) and examples in the book …

The limit cannot be determined through substitution / limit law because doesn’t exist. However, we know that and then . Since , the squeeze theorem gives , and

 Special Trig. Limits: