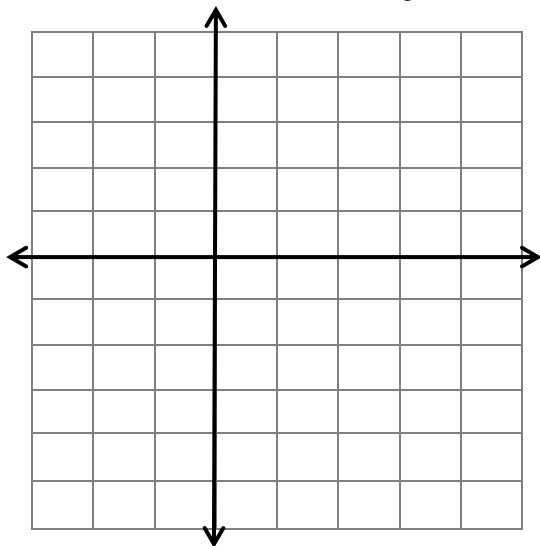


Section 1.5 (Infinite Limits)

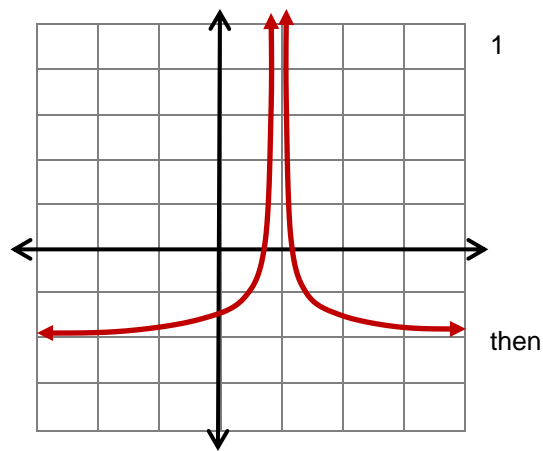
Examine the function $f(x) = \frac{2}{x-1}$... What does the graph look like? What is the domain? What happens as x approaches 1? From the left? From the right



x	$f(x) = \frac{2}{x-1}$
0	
0.5	
0.9	
0.99	
1.001	

A limit in which $f(x)$ increases or decreases without bound as x approaches c is called an infinite limit (DNE)

Example: Determine the limit of the following graph as x approaches from the left and right...



If $f(x)$ approaches \pm infinity as x approaches c , then $x = c$ is a vertical asymptote of the function.

Given **simplified** rational function $f(x) = \frac{p(x)}{q(x)}$, if c is a zero of $q(x)$,
 $x = c$ is a vertical asymptote of $f(x)$

Examples: Determine the vertical asymptotes of the following...

$$f(x) = \frac{2}{3(x+2)}$$

$$g(x) = \frac{x^2+9}{x^2-9}$$

$$h(x) = \tan(x)$$

$$f(x) = \frac{x^2+2x-15}{x^2-9}$$

Example: Determine $\lim_{x \rightarrow 1} \frac{x^2 - 2x}{x - 1}$

Consider the following properties of infinite limits when examining problems of this type...

Let c and L be real numbers and let f and g be functions such that $\lim_{x \rightarrow c} f(x) = \infty$ and $\lim_{x \rightarrow c} g(x) = L$

1. Sum or difference: $\lim_{x \rightarrow c} [f(x) + g(x)] = \infty$
2. Product: $\lim_{x \rightarrow c} [f(x)g(x)] = \infty$ (if $L > 0$ or negative infinity if $L < 0$)
3. Quotient: Sum or difference: $\lim_{x \rightarrow c} \frac{g(x)}{f(x)} = 0$

Examples: Evaluate the following limits...

$$\lim_{x \rightarrow 0} \left(4 + \frac{2}{x^2} \right)$$

$$\lim_{x \rightarrow 1^-} \frac{x^2 + 1}{\csc(x\pi)}$$

$$\lim_{x \rightarrow 0^+} [3 \cot(x)]$$

Application Example: Clark Griswold is hanging Christmas lights again when the base of his 25-ft. ladder begins to slide out from the wall of the house at a rate of 2 feet per second. The top of the ladder is moving down the wall at a rate of $r = \frac{2x}{\sqrt{x^2 - 625}}$ feet per second, where x is the distance of the base of the ladder from the wall.

- (a) Find the rate r when x is 8 feet
- (b) Find the rate r when x is 16 feet
- (c) Find the limit of r as x approaches 25 feet

