Section 2.2 (Basic Differentiation Rules and Rates of Change)

In the previous section, we used limits to determine derivatives. Now we examine differentiation rules to help find derivatives without the direct use of limits.

Examples: Use limits to find the derivatives of the following (note Pascal's triangle with time)...

f(x) = 4 f(x) = x f(x) = x^2 f(x) = x^3

Looking at the patterns from above and going through proofs (see book), we get the following rules...

Constant Rule: The derivative of a constant is $\mathbf{0}$ Power Rule: The derivative of \mathbf{x}^n is \mathbf{nx}^{n-1} (n rational)

Examples: Find the derivatives of the following...

f(x) = x g(x) = x⁵ h(x) =
$$\sqrt{x}$$
 y = $\frac{1}{x^3}$

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Example: Find the slope of the graph of $f(x) = x^3$ at x = -2, x = 0, and x = 1

<u>Example</u>: Find an equation of the tangent line to the graph of $f(x) = x^3$ when x = -2

Here are some other rules explained in more detail in the book (we'll illustrate with examples)...

Constant Multiple Rule:
$$\frac{d}{dx} [cf(x)] = cf'(x) - f(x) = cx^n$$
 implies $f'(x) = c(n-1)x^{n-1}$
Sum Rule: $\frac{d}{dx} [f(x) + g(x)] = f'(x) + g'(x)$
Difference Rule: $\frac{d}{dx} [f(x) - g(x)] = f'(x) - g'(x)$

Examples: Find the following derivatives (and get more examples from students)...

f(x) = 5x³ s(t) =
$$\frac{4t^2}{3}$$
 g(x) = $\frac{5}{3x^3}$ f(x) = 4x³ + 3x² - x + 5

Derivatives of sine and cosine:
$$\frac{d}{dx}\sin(x) = \cos(x) - \frac{d}{dx}\cos(x) = -\sin(x)$$

Examples: Find the following derivatives (and get more examples from students)...

f(x) =
$$3\cos(x)$$
 y = $\frac{\sin(x)}{3}$ g(x) = $3x^2 + 2\sin(x)$

In addition to tangent lines and slope, the derivative can also be used to determine the rate of change of one variable with respect to another (consider water flow rates, production rates, velocity, etc.). A common use is to describe the motion of an object. A function s that gives the position of an object as a function of time t is called a position function (s(t)). Or more specifically (position of a free-falling object (neglecting air resistance))

$$\mathbf{s}(\mathbf{t}) = \frac{1}{2} \mathbf{g} \mathbf{t}^2 + \mathbf{v}_0 \mathbf{t} + \mathbf{s}_0 \quad \text{--- t is time, } \mathbf{v}_0 \text{ initial velocity, } \mathbf{s}_0 \text{ initial height, and g acceleration due to gravity (32 or 9.8)}$$

With rate being distance over time (r = d/t), we get the average velocity of an object to be given by [change in distance / change in time] = $\Delta s / \Delta t$ (see better explanation in book)...

<u>Application Example</u>: At time t = 0, the Hokie bird throws a football from a 32-foot platform diving board with an initial velocity of 16 feet per second.

- (a) See sketch for example 10 in the book
- (b) Find the position function of the ball
- (c) At what time (t) will the ball hit the ground?
- (d) What is the ball's velocity at impact?

--- DO HOMEWORK EXAMPLES WITH TIME ---