**Section 2.3** (Product and Quotient Rules and Higher Order Derivatives)

We covered the sum and difference rules for derivatives in the previous section. The product and quotient rules are a little more involved (see book for proofs)…

1. Product Rule: $\frac{d}{dx}$ **[f(x)g(x)] = f(x)g`(x) + g(x)f `(x)** (note: not the same as a composition function)

--- 1st times the derivative of the 2nd + 2nd times the derivative of the 1st ---

1. Quotient Rule: $\frac{d}{dx} \left[\frac{f(x)}{g(x)}\right]$ **=** $\frac{g\left(x\right)f^{'}\left(x\right)-f\left(x\right)g'(x)}{[g\left(x\right)]^{2}}$

--- low-d-hi minus hi-d-low over low squared ---

Examples: Find the following derivatives

h(x) = (5x2 – 2x)(3x + 4) y = 4x3 sin(x) y = 3x sin(x) – 4 cos(x) f(x) = x3 sin(x) cos(x)

h(x) = $\frac{5x+2}{x^{2}-7}$ y = $\frac{6-\left(^{1}/\_{x}\right)}{x+9}$ h(x) = $\frac{x^{3}-4x}{5}$

Knowing the derivatives of sin and cos allows us to find the derivatives of the remaining trigonometric functions.

Example: Use the quotient rule to find the derivative of tan(x)

Derivatives of trigonometric functions:

 $\frac{d}{dx}$ sin(x) = cos(x) $\frac{d}{dx}$ cos(x) = – sin(x) $\frac{d}{dx}$ tan(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

$\frac{d}{dx}$ csc(x) = – csc(x)cot(x) $\frac{d}{dx}$ sec(x) = sec(x)tan(x) $\frac{d}{dx}$ cot (x) = – csc2(x)

Examples: Find the derivatives of the following…

y = 3x5 – cot(x) y = x2 csc(x) y = $\frac{1-cos⁡(x)}{sin⁡(x)}$ = csc(x) – cot(x)

--- Note that much of the work in finding derivatives is in simplifying after differentiation ---

Similarly to finding velocity with the derivative of the position function (velocity = change in position over time), we can obtain acceleration using the derivative of the velocity function (acceleration = change in velocity over time).

 Position: s(t) Velocity: v(t) = s`(t) Acceleration: a(t) = v`(t) = s``(t)

The function a(t) is the second derivative of s(t) --- This is the derivative of the first derivative (other higher-order derivatives follow)

Application Example: (Review example 10 in the book) Paul Blart is travelling at a rate of 22.5 mph (33 feet per second) on his Segway when the brakes are applied. The position function of the Segway is s(t) = – 4.3t2 + 22.5t, where s is measured in feet and t in seconds. Use this function to complete the table…



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| t | 0 | 1 | 2 | 3 | 4 |
| s(t) |  |  |  |  |  |
| v(t) |  |  |  |  |  |
| a(t) |  |  |  |  |  |