Section 2.3 (Product and Quotient Rules and Higher Order Derivatives)

We covered the sum and difference rules for derivatives in the previous section. The product and quotient rules are a little more involved (see book for proofs)…

1. Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g`(x) + g(x)f`(x)$ (note: not the same as a composition function)

   --- 1st times the derivative of the 2nd + 2nd times the derivative of the 1st ---

2. Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

   --- low-d-hi minus hi-d-low over low squared ---

Examples: Find the following derivatives

$h(x) = (5x^2 - 2x)(3x + 4)$ $y = 4x^3 \sin(x)$ $y = 3x \sin(x) - 4 \cos(x)$ $f(x) = x^3 \sin(x) \cos(x)$

$h(x) = \frac{5x+2}{x^2-7}$ $y = \frac{6-(1/x)}{x+9}$ $h(x) = \frac{x^3-4x}{5}$

Knowing the derivatives of sin and cos allows us to find the derivatives of the remaining trigonometric functions.

Example: Use the quotient rule to find the derivative of tan(x)

Homework problems: 2,3,5,8,9,11,14,25,27,33,34,42,43,73,93,98,118 (required) 53,74,84,87,92,100,102 (recommended)

Derivatives of trigonometric functions:

$\frac{d}{dx}\sin(x) = \cos(x)$	$\frac{d}{dx}\cos(x) = -\sin(x)$	$\frac{d}{dx}\tan(x) =$ _____
$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$	$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$	$\frac{d}{dx}\cot(x) = -\csc^2(x)$

<u>Examples</u>: Find the derivatives of the following…

$y = 3x^5 - \cot(x)$ $\qquad$ $y = x^2\csc(x)$ $\qquad$ $y = \frac{1-\cos(x)}{\sin(x)} = \csc(x) - \cot(x)$

--- Note that much of the work in finding derivatives is in simplifying after differentiation ---

Similarly to finding velocity with the derivative of the position function (velocity = change in position over time), we can obtain acceleration using the derivative of the velocity function (acceleration = change in velocity over time).

Position: $s(t)$ $\qquad$ Velocity: $v(t) = s`(t)$ $\qquad$ Acceleration: $a(t) = v`(t) = s``(t)$

The function a(t) is the second derivative of s(t) --- This is the derivative of the first derivative (other higher-order derivatives follow)

Application Example: (Review example 10 in the book) Paul Blart is travelling at a rate of 22.5 mph (33 feet per second) on his Segway when the brakes are applied. The position function of the Segway is $s(t) = -4.3t^2 + 22.5t$, where s is measured in feet and t in seconds. Use this function to complete the table…

t	0	1	2	3	4
s(t)					
v(t)					
a(t)					

Homework problems: 2,3,5,8,9,11,14,25,27,33,34,42,43,73,93,98,118 (required) 53,74,84,87,92,100,102 (recommended)