Section 2.4 (The Chain Rule)

Composite functions can be more complex and we have not yet talked about their derivatives.

Example: Find the derivative of $h(x) = (2x^3 - 4)^2$ using formulas covered to this point.

Example: Decompose the previous into 2 functions f(x) and g(x) such that h(x) = f(g(x))

Chain Rule: If y = f(u) is a differentiable function of u and u = f(x) is a differentiable function of x, then y = f(g(x)) is a differentiable function of x and $\frac{d}{dx}$ **[f(g(x))] = f \`(g(x)) g\`(x)** or $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

--- For composite functions, take the derivative of the outer function and multiply by the derivative of the inner function ---

Example: Find the derivative of $h(x) = (2x^3 - 4)^2$ using the chain rule

Examples: Decompose the following composite functions y = f(g(x)) into smaller functions u = g(x) and y = f(u)

$y = (3x + 8)^4$ $y = \frac{3}{2x+5}$ $y = \cos(3x)$ $y = \sqrt{3x^3 - x + 5}$ $y = \sin^3(x)$

As a result of the chain rule we can generalize the power rule to be $\frac{d}{dx}[u^n] = nu^{n-1}u'$

Example: Find the derivative of $f(x) = (3x + 8)^4$

Homework problems: 1,2,3,7,8,17,20,27,29,33,45,48,51,53,65,111 (required) 14,18,43,67,70,115 (recommended)

Example: Find the derivative of f(x) = $\sqrt[3]{(x^2 - 1)}$

Where does f `(x) = 0 and where does it not exist?

Review examples 7,8,9 in the book and work some of the homework exercises 7-36

The chain rule also applies to derivatives of trigonometric functions…

$\frac{d}{dx}$ [sin u] = (cos u) u`      $\frac{d}{dx}$ [cos u] = – (sin u) u`   …

Examples: Find the derivatives of the following functions…

y = sin(4x)   y = $\sin^2(x)$ y = $\sqrt{\sin(x)}$ y = $\sin^2(8\theta)$

--- Note that if a derivative involves the composition of more than 2 functions you may need to apply the chain rule more than once

Homework problems: 1,2,3,7,8,17,20,27,29,33,45,48,51,53,65,111 (required) 14,18,43,67,70,115 (recommended)