Section 2.5 (Implicit Differentiation)

Up to this point we've looked at equations in which y is expressed explicitly as a function of x ($y = f(x) = x^2 + 3$ for example). What happens when we are unable to write y explicitly as a function of x ($x^2 - 2y^3 + 4y = 2$)?

PARENTAL ADVISORY EXPLICIT CONTENT

Example: Use implicit differentiation to differentiate xy^2 with respect to x…

$$\frac{d}{dx}[xy^2] = x\frac{d}{dx}[y^2] + y^2\frac{d}{dx}[x] \quad \text{--- Product rule}$$
$$= x\left[2y\frac{dy}{dx}\right] + y^2(1) \quad \text{--- Chain rule}$$
$$= 2xy\frac{dy}{dx} + y^2 \quad \text{--- Simplify}$$

Guidelines for implicit differentiation…

1. Differentiate both sides of the equation *with respect to x*.
2. Collect all terms involving dy/dx on the left side and move other terms to right side
3. Factor dy/dx out of the left side of the equation
4. Solve for dy/dx

Example: Find dy/dx given $y^3 + y^2 - 3y - x^2 = 5$

Example: Determine the slope of the graph of $(x^2 + 4)y = 8$ at point $(-2, 1)$

Example: Determine the slope of the graph of $3(x^2 + y^2) = 100xy$ at the point $(3, 1)$

Homework problems: 1,2,5,7,11,21,22,26,29,33,35,39,45 (required) 17,23,32,34,59,71 (recommended)

Example: Find an equation of the tangent line to $(x + 2)^2 + (y - 3)^2 = 25$ at (3,3) (sketch if time)

Example: For equation $x^2 + y^2 - 4x + 6y + 9 = 0$, find the following…

1. Find function(s) by solving the equation for y in terms of x
2. Sketch the graph(s)
3. Differentiate the explicit function(s)
4. Use implicit differentiation on the original function and show the results are equivalent

Sometimes you can use the original equation or derivative to find and simplify higher-order derivatives

Example: Find $\frac{d^2y}{d^2x}$ of $x^2y^2 - 2x = 3$ in terms of x and y

Review Examples 3 and 6 in the book and HW number 71 with time

Homework problems: 1,2,5,7,11,21,22,26,29,33,35,39,45 (required) 17,23,32,34,59,71 (recommended)