## Section 2.6 (Related Rates)

When water is drained from a cylindrical tank, the volume V , the radius r , and the height h of the water level are all functions of time $t$. These variables are related by the equation $V=\pi r^{2} h$. You can differentiate implicitly to obtain the related rate equation...

$$
\left.\frac{d}{d t}(\mathrm{~V})=\frac{d}{d t}\left(\pi \mathrm{r}^{2} \mathrm{~h}\right)=\pi\left[\mathrm{r}^{2} \frac{d h}{d t}+\mathrm{h}\left(2 \mathrm{r} \frac{d r}{d t}\right)\right]=\pi\left[\mathrm{r}^{2} \frac{d h}{d t}+2 \mathrm{rh} \frac{d r}{d t}\right)\right],
$$

Showing that the rate of change in volume is related to the rates of change in both $h$ and $r$.
Example: Suppose x and y are both differentiable functions of t and are related by the equation $\mathrm{y}=3 \mathrm{x}^{2}-4$. Find $d y / d t$ when $x=2$, given that $d x / d t=4$ when $x=2$.

In the previous example, you were essentially given the equations and values. In remaining examples, you will need to create the mathematical models to be used.

Example: The rocket city rednecks have set up a television camera at ground level to file the lift-off of a new Space-X rocket. The position equation of the rocket's rise is estimated to be $s=60 t^{2}$, where $s$ is measured in feet and $t$ in seconds. The camera is on level ground and placed 2200 feet from the launch pad. Find the rate of change in the angle of elevation of the camera at $t=5$ seconds and $t=10$ seconds.


Example: Ernie does a cannonball into a still pool of water, forming ripples in the form of concentric circles. The radius $r$ of the outer ripple is increasing at a rate of 3 feet per second. When the radius is 9 feet, at what rate is the total area A of the water changing?


Example: An airplane is flying on a flight path that will take it directly over a radar tracking station (see example 4 in the book). If the ground range is decreasing at a rate of 300 miles per hour (assume level ground) when the slant range is 12 miles, what is the speed of the plane if the plane flies at a constant altitude of 5 miles?

--- Complete homework exercises and get examples from students / other sources with time ---

