## Section 3.4 (Concavity and the Second Derivative Test)

Examine the slopes of the tangent lines in the following graph of a concave down function.
Is there a critical point? relative maximum / minimum?
Are the slopes increasing or decreasing?
If $\mathbf{f}^{\prime \prime}(\mathbf{x})>\mathbf{0}$ for all $\mathbf{x}$ in a given interval ( $2^{\text {nd }}$ derivative is positive, meaning that the slopes of the tangent lines are increasing), then $f(x)$ is concave up on that interval.

If $\mathbf{f}^{\prime}{ }^{\prime}(\mathbf{x})<\mathbf{0}$ for all x in a given interval ( $2^{\text {nd }}$ derivative is negative, meaning that the slopes of the tangent lines are decreasing), then $f(x)$ is concave down on that interval.


What if $f^{\prime \prime}(x)=0$ ?
Example: Determine open intervals on which the graph of $f(x)=x^{3}-3 x^{2}+1$ is concave up or concave down.

Example: Determine open intervals on which the graph of $\mathrm{f}(\mathrm{x})=\frac{x^{2}+1}{x^{2}-4}$ is concave up or concave down.

A point of inflection on the graph of $f$ is a point $\left(f^{\prime}(x)=0\right)$ where concavity changes from down to up (or vice versa) Example: Find the points of inflection and discuss concavity of $f(x)=2 x^{4}-8 x+3$

## Second derivative test when $\mathrm{f}^{`}(\mathrm{c})=0$ and $\mathrm{f}^{\prime}$ (c) exists:

1. If $f$ " $(\mathrm{c})>0$, then $f$ has a relative minimum at $(\mathrm{c}, \mathrm{f}(\mathrm{c}))$
2. If $f^{\prime \prime}(\mathrm{c})<0$, then $f$ has a relative maximum at ( $\mathrm{c}, \mathrm{f}(\mathrm{c})$ )

Example: Find all relative extrema of $f(x)=x^{3}-5 x^{2}+7 x$

